

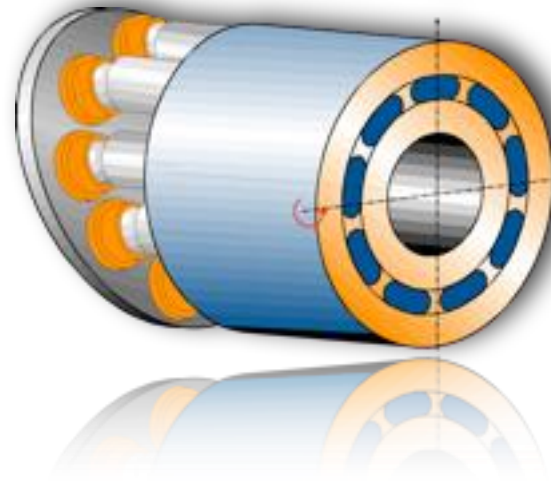
# Αρχές λειτουργίας και τεχνολογία μηχανών θετικής μετατόπισης

Σ. Τσολάκης

ΕΜΠ  
24/02/2020

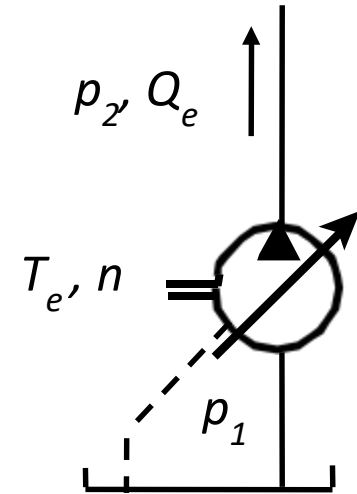
# Περιεχόμενα

- Displacement machines – design principles & scaling laws
- Power density comparison between hydrostatic and electric machines
- Volumetric losses, effective flow, flow ripple, flow pulsation
- Steady state characteristics of an ideal and real displacement machine
- Torque losses, torque efficiency

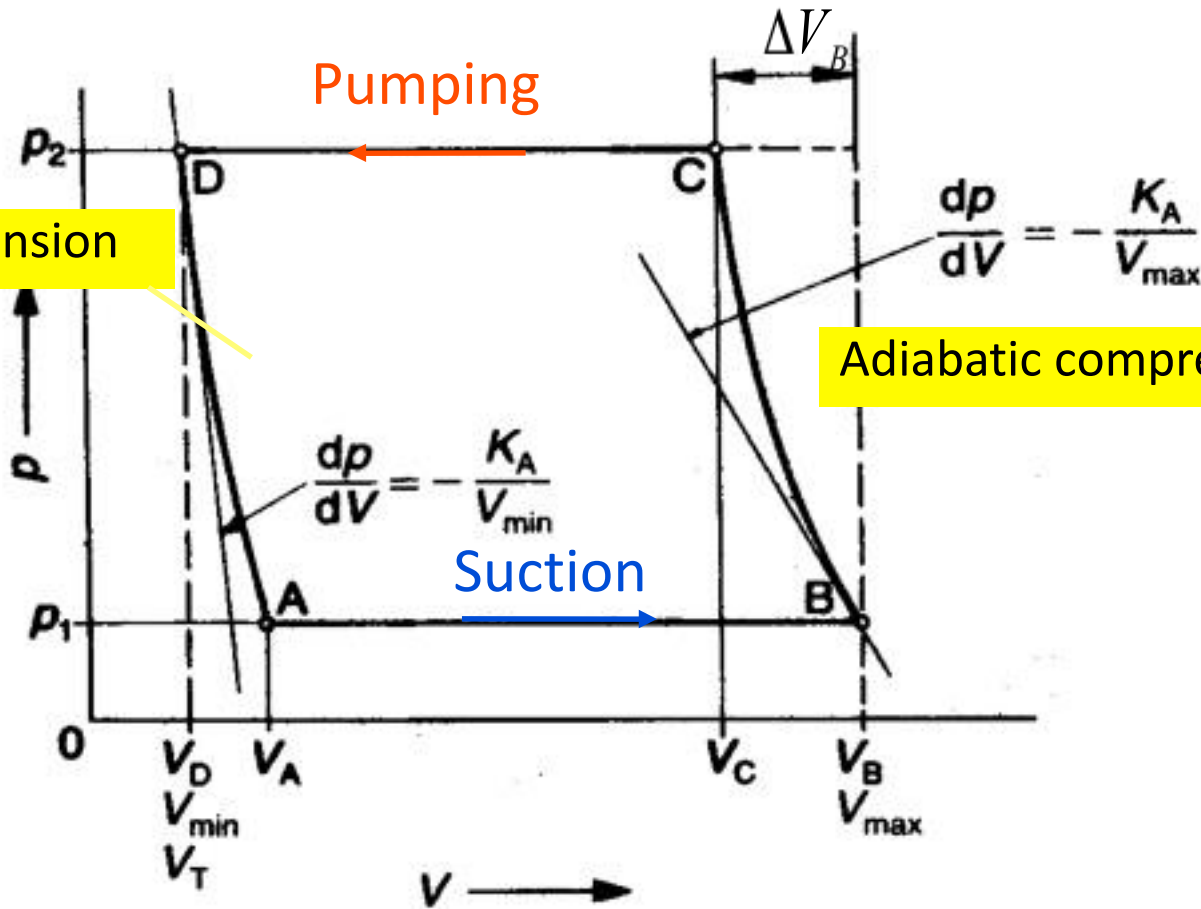


# Μηχανή θετικής μετατόπισης

due to compressibility of a real fluid



Adiabatic expansion



Adiabatic compression

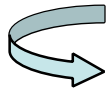
$V_{\min} = V_T$  with  $V_T$ .. dead volume

$K_{A..}$  adiabatic bulk modulus

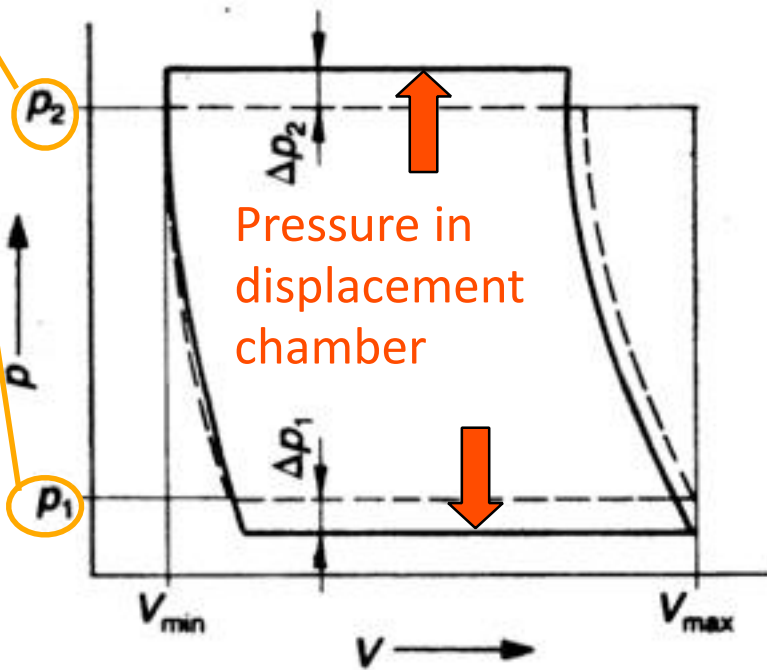
# Μηχανή θετικής μετατόπισης

due to viscosity & compressibility of a real fluid

Pressure drop between displacement chamber and port

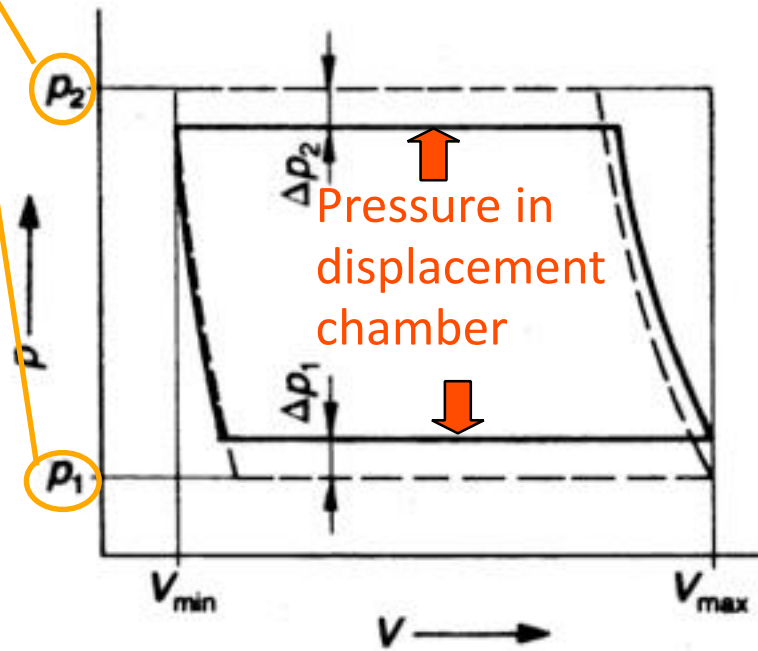


Port pressure



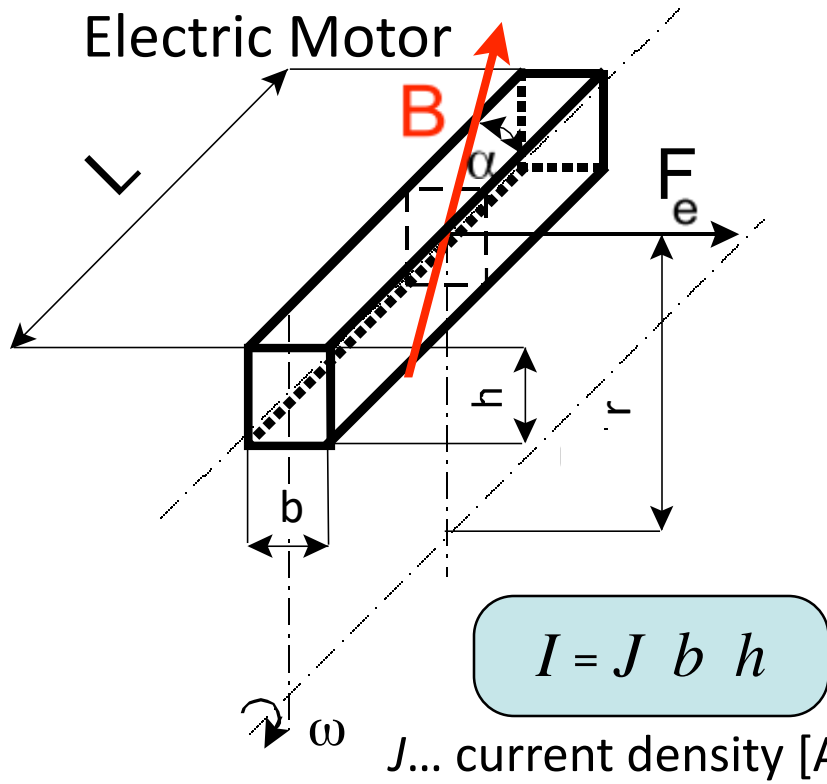
Pump

Port pressure



Motor

# Συγκέντρωση ισχύος

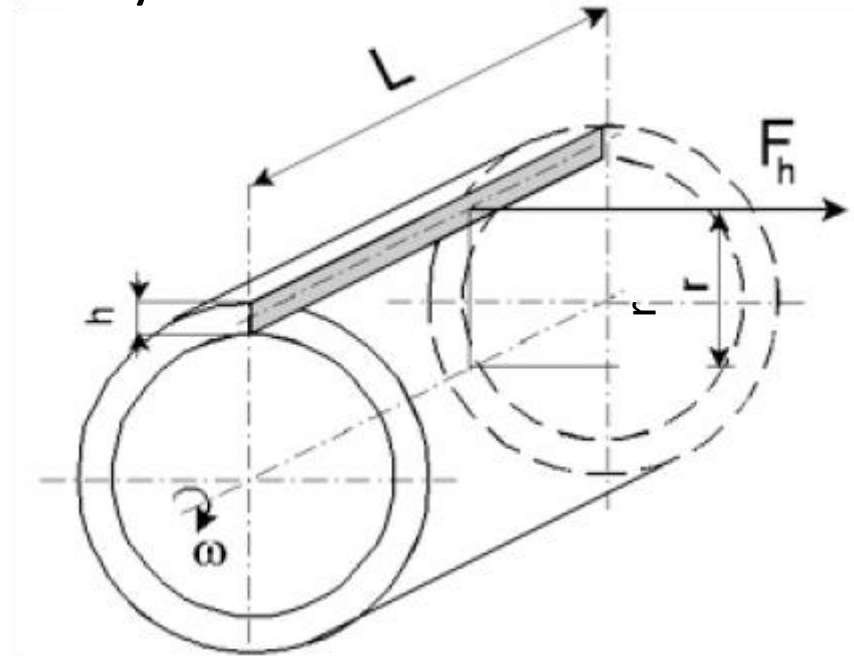


$$F_e = I \cdot B \cdot L \cdot \sin \alpha \quad \text{with } I \text{ current [A]}$$

$B \dots$  magnetic flux density [ T ] or [Vs/m<sup>2</sup>]

Torque:  $T = I \cdot B \cdot L \cdot r \cdot \sin \alpha$

Hydraulic Motor



$$F_h = p \cdot L \cdot h$$

$$T = p \cdot L \cdot h \cdot r$$

# Παράδειγμα

Power:  $P = T \omega = T 2\pi n$

For electric motor follows:  $P = I B L r 2\pi n$  assuming  $\alpha=90^\circ$

For hydraulic motor follows:  $P = p \cdot L \cdot h \cdot r \cdot 2\pi \cdot n$

Force density: Electric Motor

Hydraulic Motor

$$\frac{F_e}{L \cdot h} = \frac{J \cdot b \cdot h \cdot B \cdot L}{L \cdot h} = J \cdot b \cdot B$$

$$\frac{F_h}{L \cdot h} = p$$

$7.6 \cdot 10^6 \text{ A} \cdot \text{m}^2 \cdot 1.8 \text{ Vs} \cdot \text{m}^{-2} \cdot 3 \cdot 10^{-3} \text{ m} = 4.1 \cdot 10^4 \text{ Pa}$

up to  $5 \cdot 10^7 \text{ Pa}$

with a cross section area of conductor:  $9 \cdot 10^{-6} \text{ m}^2$

# Λόγος μάζας-ισχύος

Electric Machine

Positive displacement machine

$$\frac{\text{mass}}{\text{power}} = 1 \dots 15 \text{ kg/kW}$$

$$0.1 \dots 1 \text{ kg/kW}$$



Positive displacement machines (pumps & motors) are:

- 10 times lighter
- min. 10 times smaller
- much smaller mass moment of inertia (approx. 70 times)



much better dynamic behavior of displacement machines

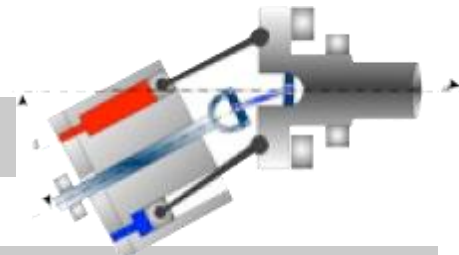
# Μηχανές θετικής μετατόπισης

Piston Machines

Axial Piston Machines



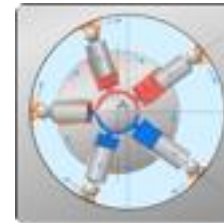
Swash Plate Machines



Bent Axis machines

In-line Piston Machines

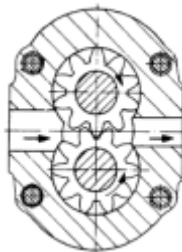
Radial Piston Machines



with external piston support

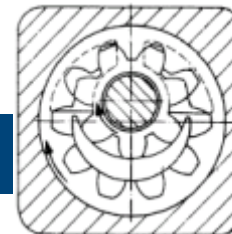
with internal piston support

Gear Machines

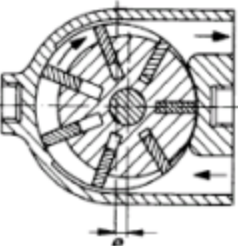
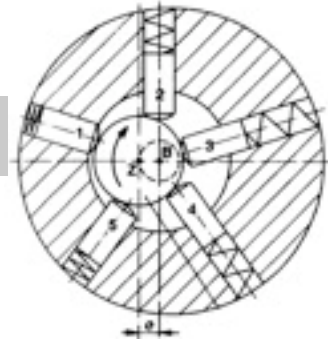


External Gear

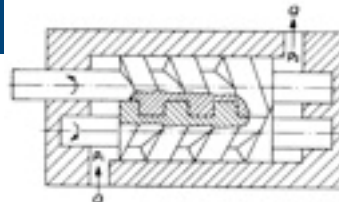
Internal Gear



Annual Gear



Vane Machines



Screw Machines

others

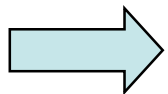
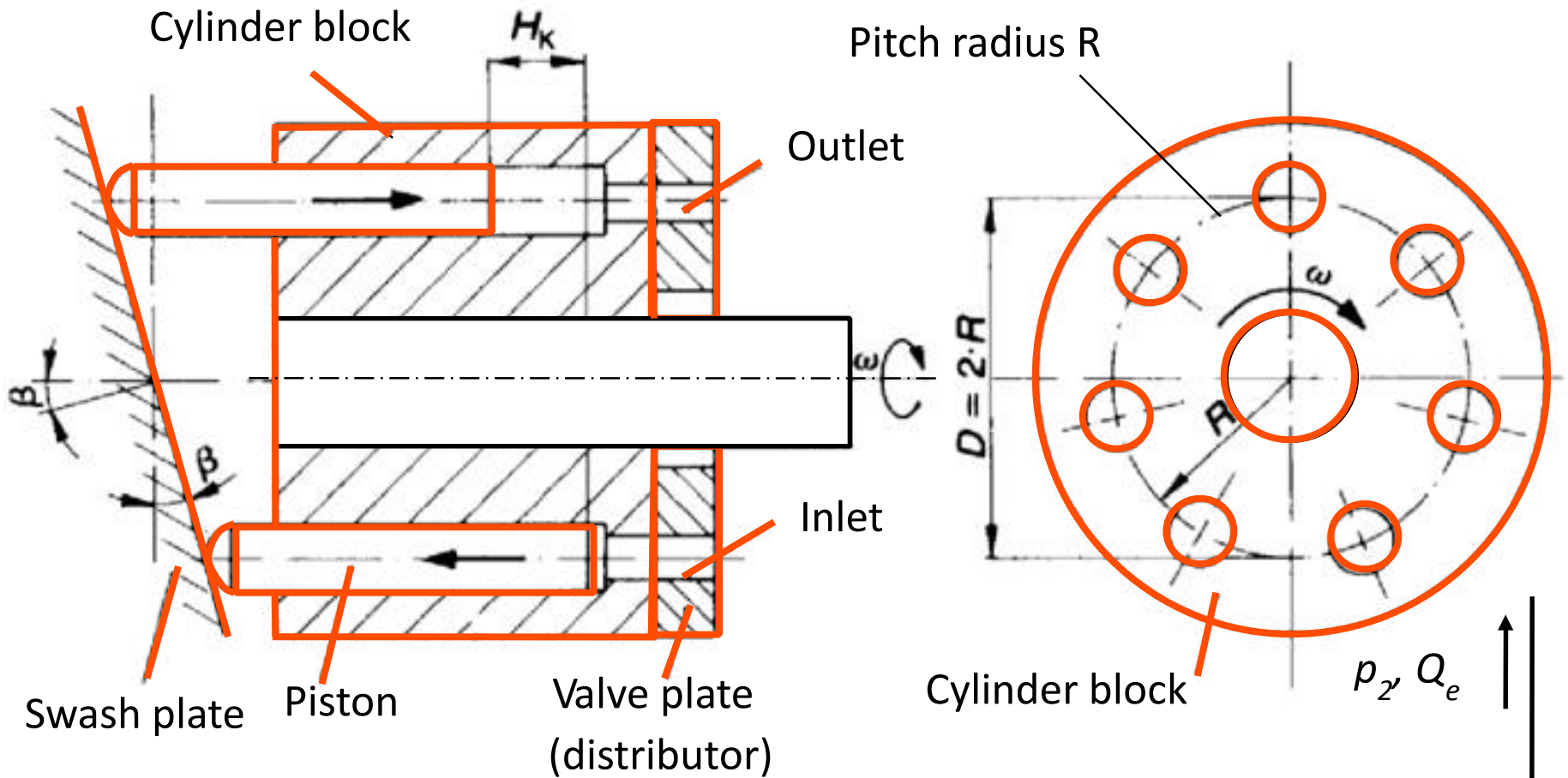
Fixed displacement machines



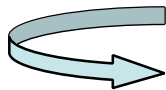
Variable displacement machines



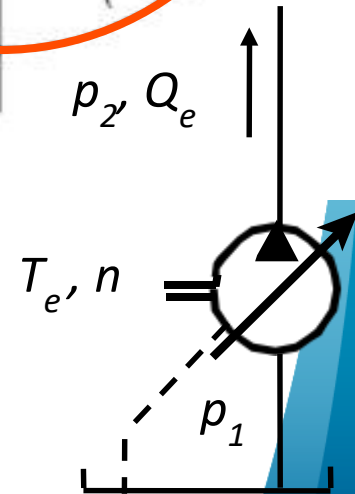
# Αντλίες αξονικών εμβόλων



Piston stroke =  $f(\beta, R)$



Variable displacement pump  
Requires continuous change of  $\beta$

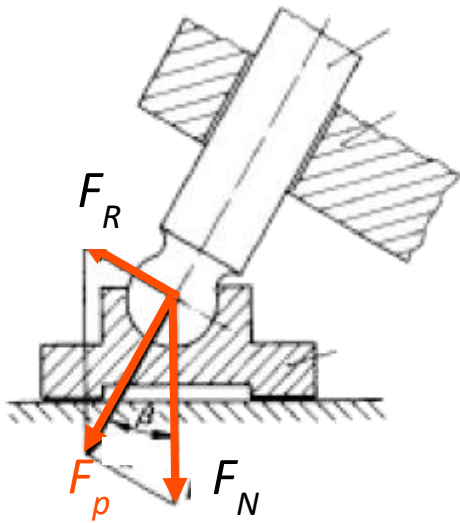


# Μηχανές Bent Axis & Swash Plate

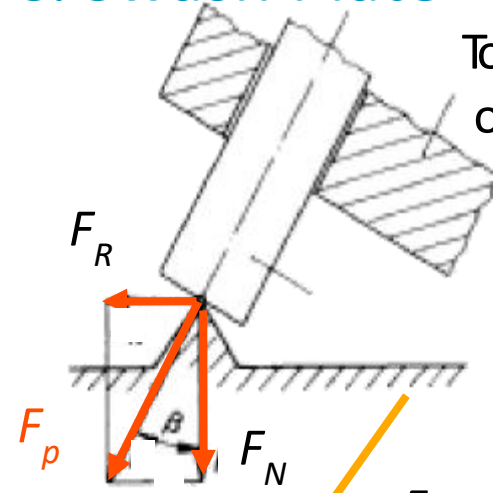
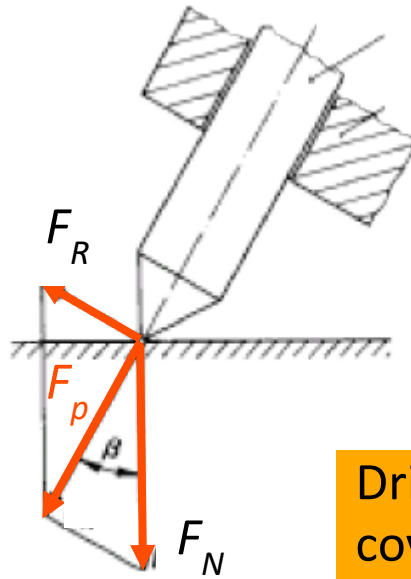
Torque generation on cylinder block

Swash plate design

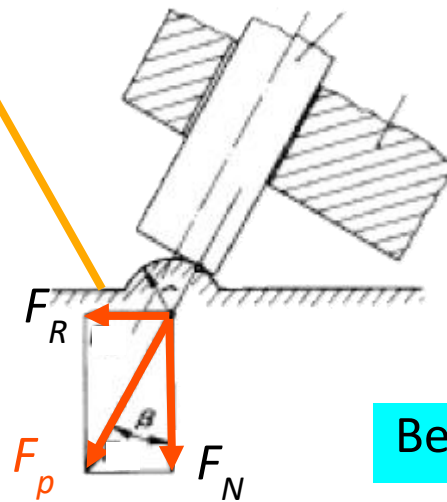
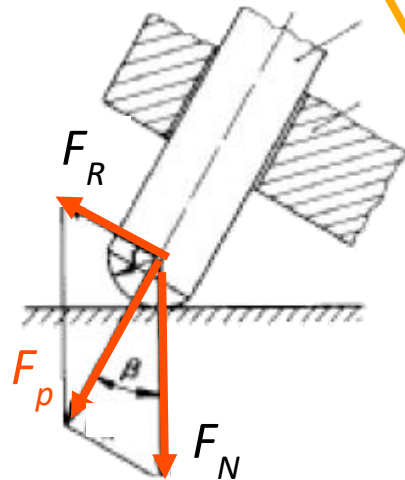
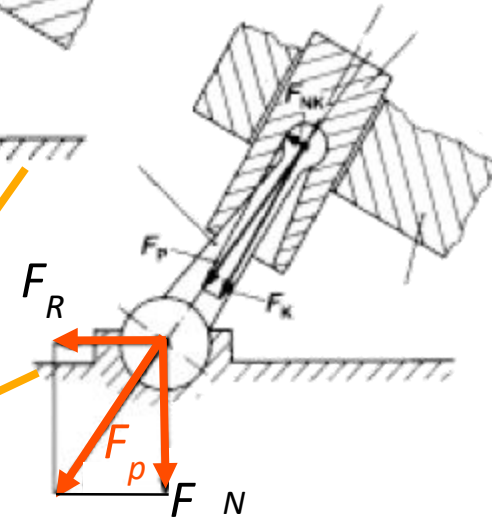
Torque generation on swash plate



Radial force  $F_R$  exerted on piston!



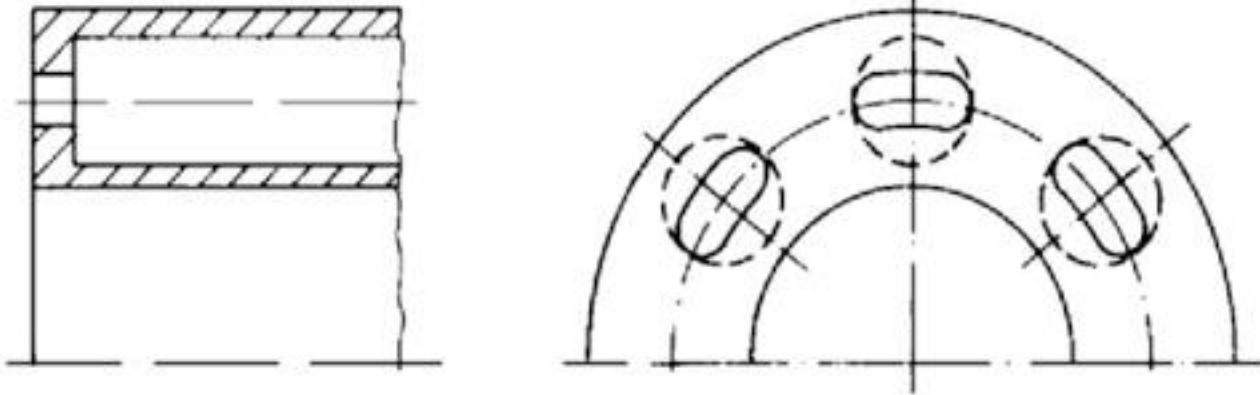
Driving flange must cover radial force



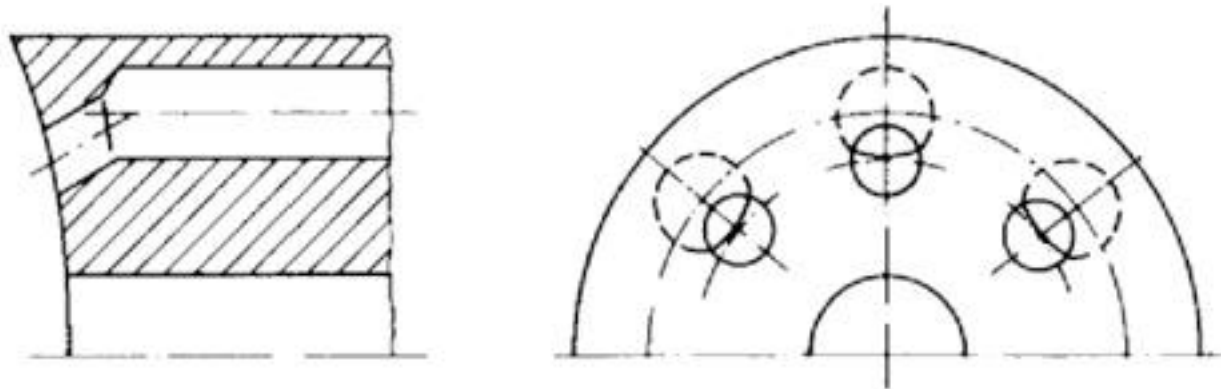
Bent axis machines

# Αντλίες αξονικών εμβόλων

Openings in cylinder bottom In case of plane valve plate



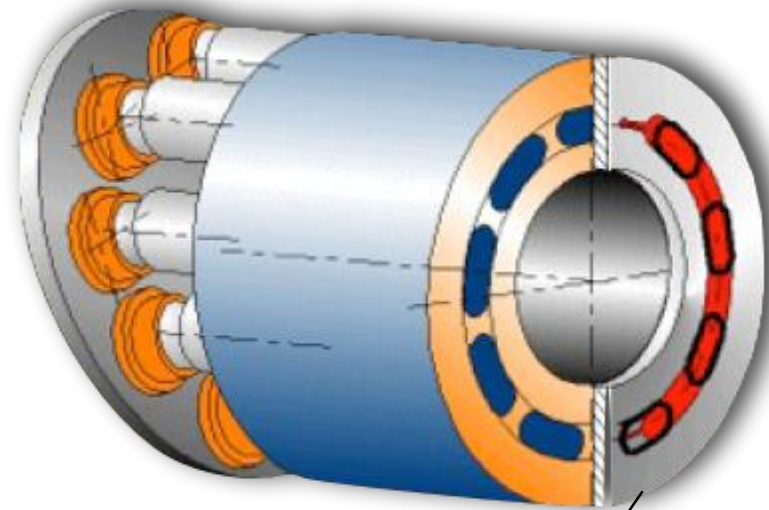
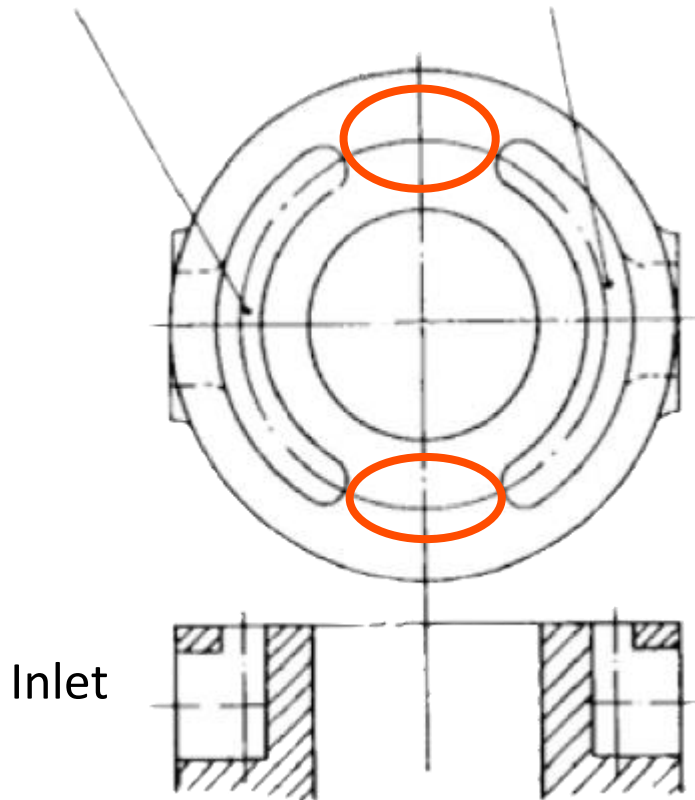
In case of spherical valve plate



# Αντλίες αξονικών εμβόλων

Plane valve plate  
Inlet opening

Outlet opening



Plane valve plate



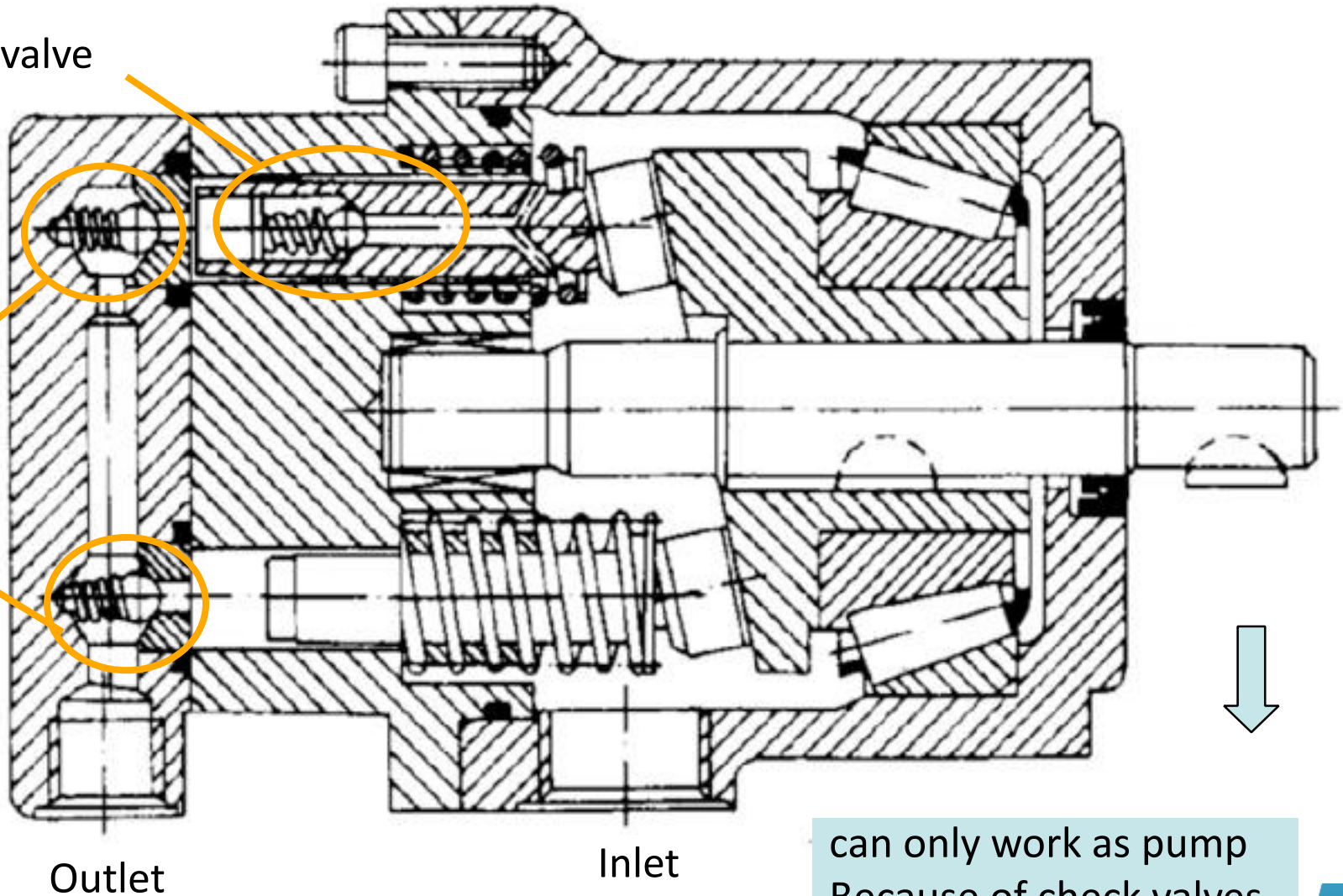
Connection of displacement chambers with suction and pressure port

# Αντλίες αξονικών εμβόλων

Kinematic reversal: pump with rotating swash plate

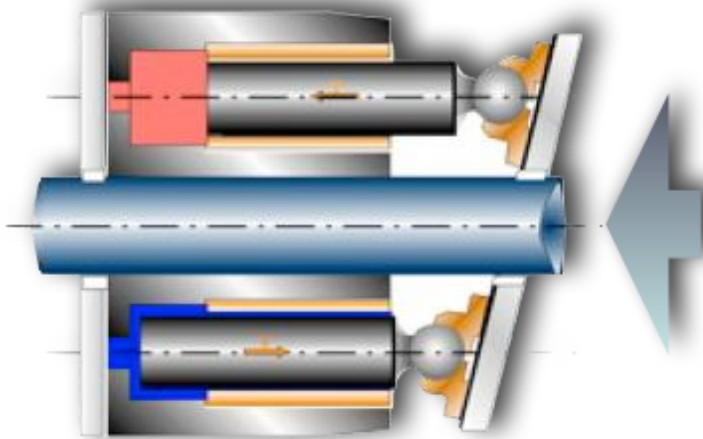
Suction valve

Pressure valve for each cylinder



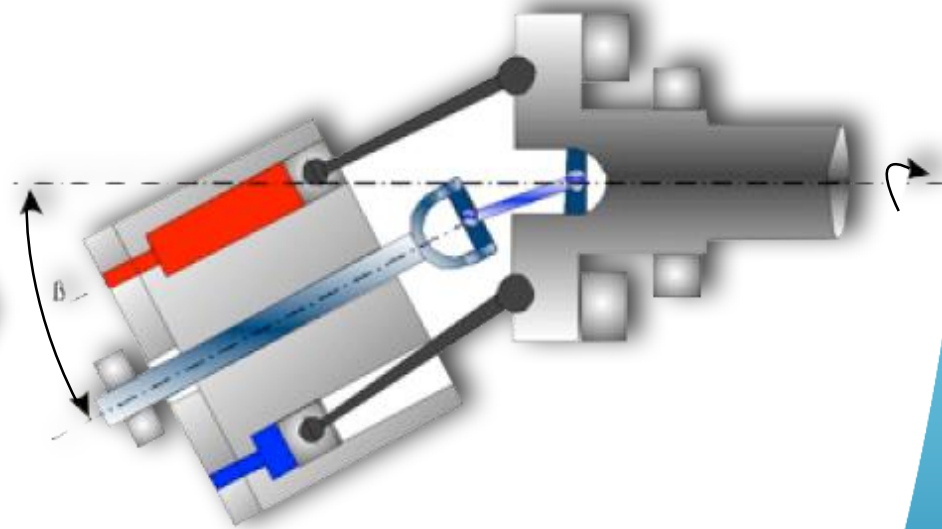
can only work as pump  
Because of check valves

# Αντλίες αξονικών εμβόλων-Σύγκριση



- Simple and compact design
- Short response time, high bandwidth
- Through going shaft
- Long service life, low loaded bearings
- Limited swash plate angle  $\beta_{\max}$  ca.  $21^\circ$
- High radial piston forces

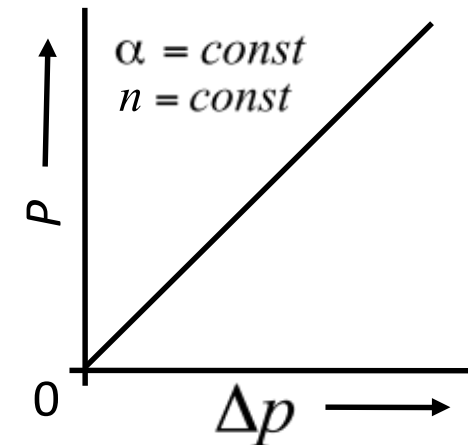
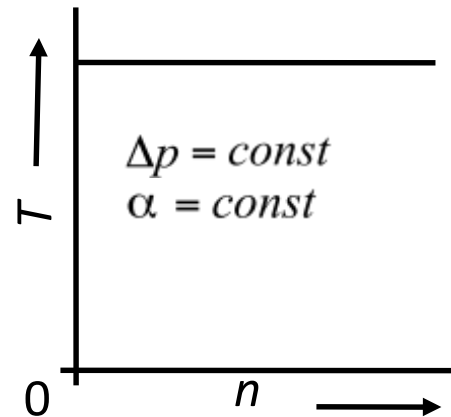
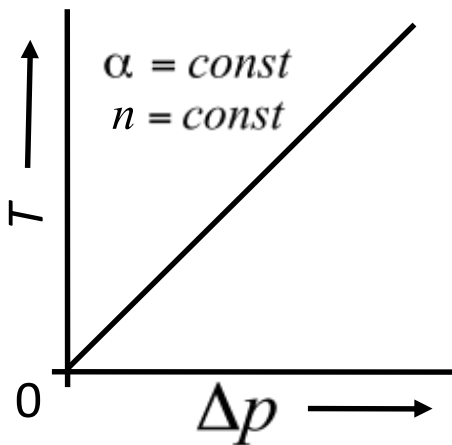
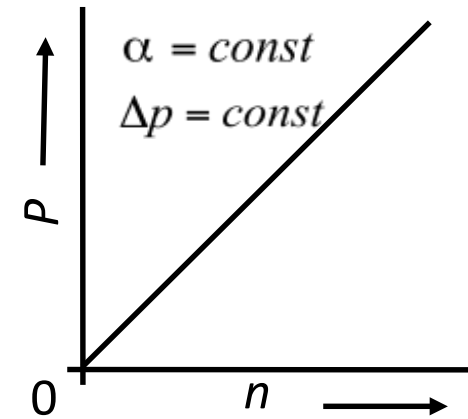
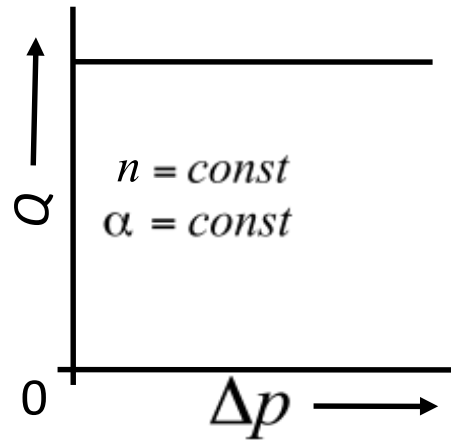
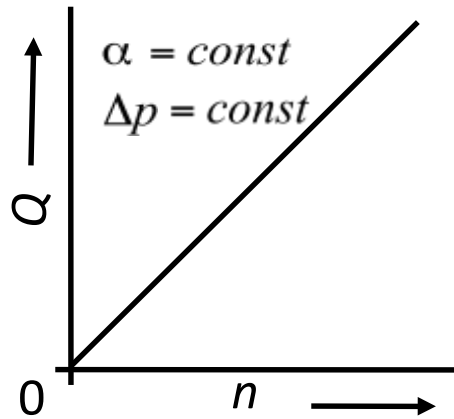
- higher max. speed
- Angle  $\beta$  up to  $45^\circ$
- Less losses
- High loaded bearings
- Expensive design
- Synchronisation required



# Συμπεριφορά steady state

Displacement volume of a variable displacement machine:

$$V = \alpha V_{max}$$



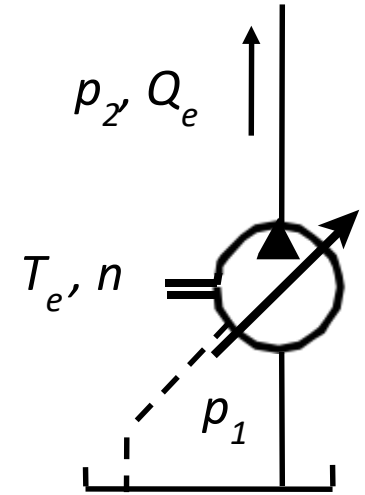
# Ομοιότητα

The pump size is determined by the displacement volume  $V$  [ $\text{cm}^3/\text{rev}$ ]. Usually a proportional scaling law, conserving geometric similarity, is applied, resulting in stresses remaining constant for all sizes of units.

$$T = \frac{\Delta p \cdot V}{2 \cdot \pi}$$

$$Q = V n$$

$$\Delta p = p_2 - p_1$$



First Order Scaling Laws :  $\lambda$  ... linear scaling factor

$$L = \lambda \cdot L_0$$

$$T = \lambda^3 \cdot T_0$$

$$P = \lambda^2 \cdot P_0$$

$$V_i = \lambda^3 \cdot V_{i0}$$

$$m = \lambda^3 \cdot m_0$$

$$n = \lambda^{-1} \cdot n_0$$

Assuming same maximal operating pressures for all unit sizes and a constant maximal sliding velocity !



## Παράδειγμα

The maximal shaft speed of a given pump is 5000 rpm. The displacement volume of this pump is  $V = 40 \text{ cm}^3/\text{rev}$ . The maximal working pressure is given with 40 MPa. Using first order scaling laws, determine:

- the maximal shaft speed of a pump with  $90 \text{ cm}^3/\text{rev}$
- the torque of this larger pump
- the maximal volume flow rate of this larger pump
- the power of this larger pump

For the linear scaling factor follows:

$$\lambda = \sqrt[3]{\frac{V}{V_0}} = \sqrt[3]{\frac{90}{40}} = 1.31$$

Maximal shaft speed of the larger pump:  $n = \lambda^{-1} \cdot n_0 = 1.31^{-1} \cdot 5000 \text{ rpm} = 3816.8 \text{ rpm}$

Torque of the larger pump:

$$T = \frac{\Delta p \cdot V}{2 \cdot \pi} = \frac{40 \cdot 10^6 \text{ Pa} \cdot 90 \cdot 10^{-6} \text{ m}^3}{2 \cdot \pi} = 573.25 \text{ Nm}$$

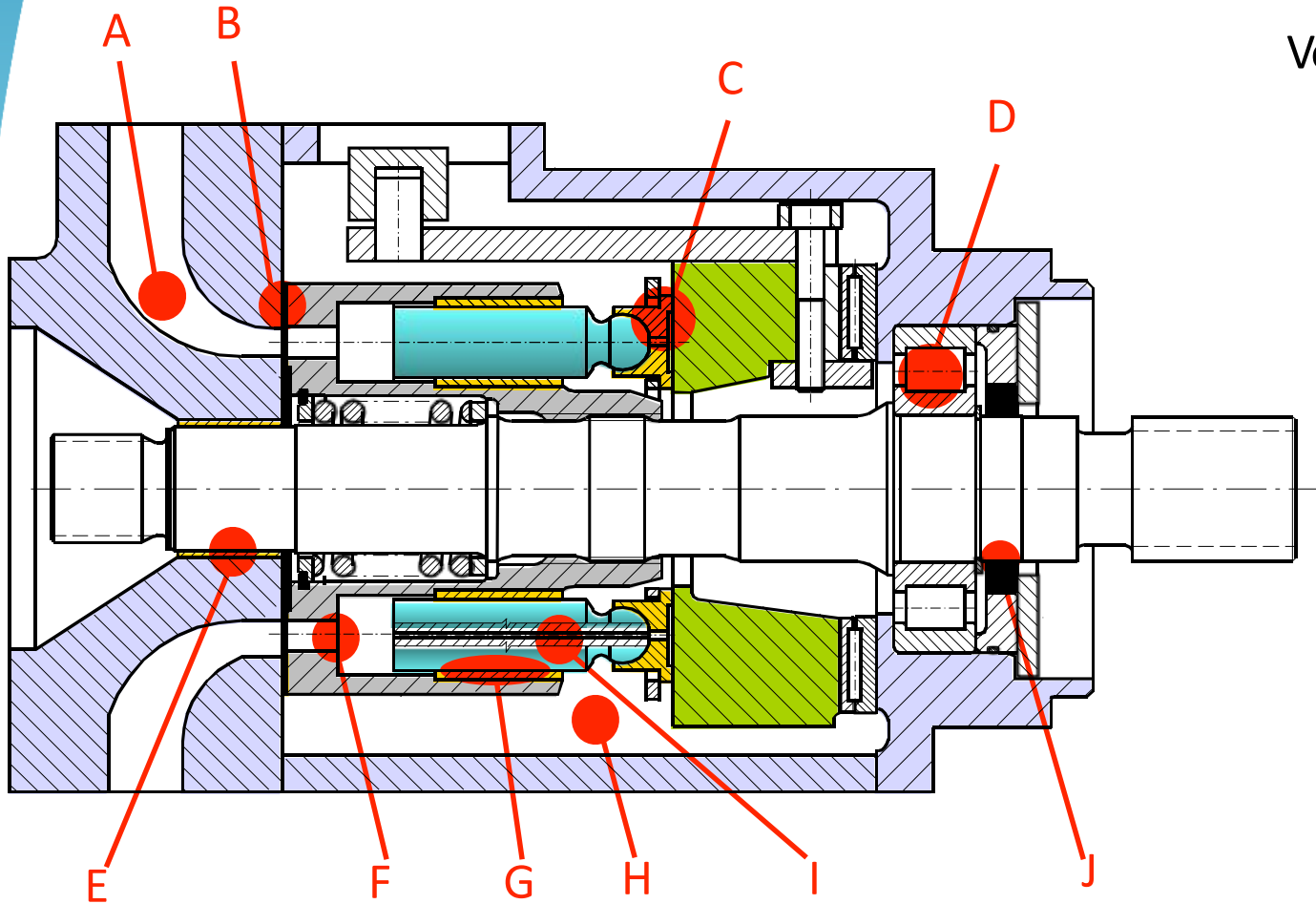
Maximal volume flow rate:

$$Q_{\max} = V \cdot n_{\max} = 90 \cdot 10^{-6} \text{ m}^3/\text{rev} \cdot 3816.8 \text{ rpm} = 0.3435 \text{ m}^3/\text{min} = 343.5 \text{ l/min}$$

Power of the larger pump:  $P = \Delta p \cdot Q = 40 \cdot 10^6 \text{ Pa} \cdot 0.3435 \text{ m}^3 \cdot \frac{1}{60} \text{ s}^{-1} = 229 \text{ kW}$

# Απώλειες στις αντλίες θετικής μετατόπισης

Axial piston machine – swash plate design



Volumetric Losses

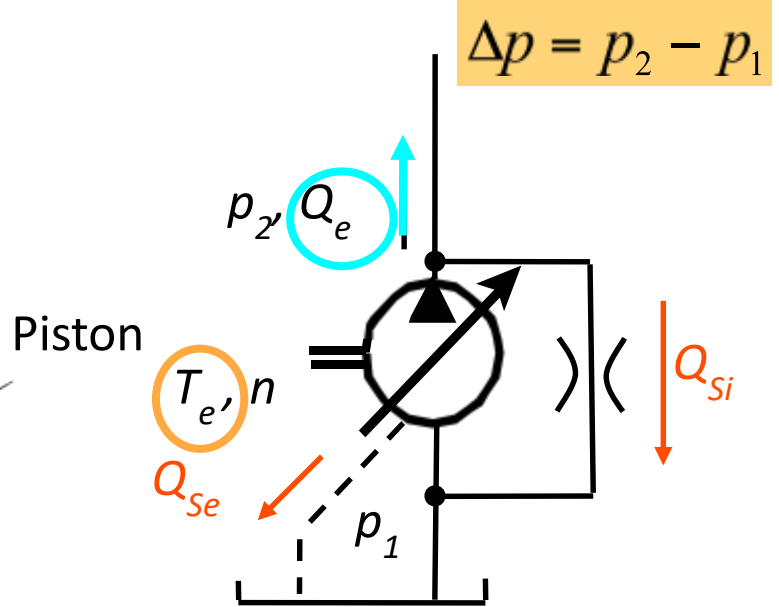
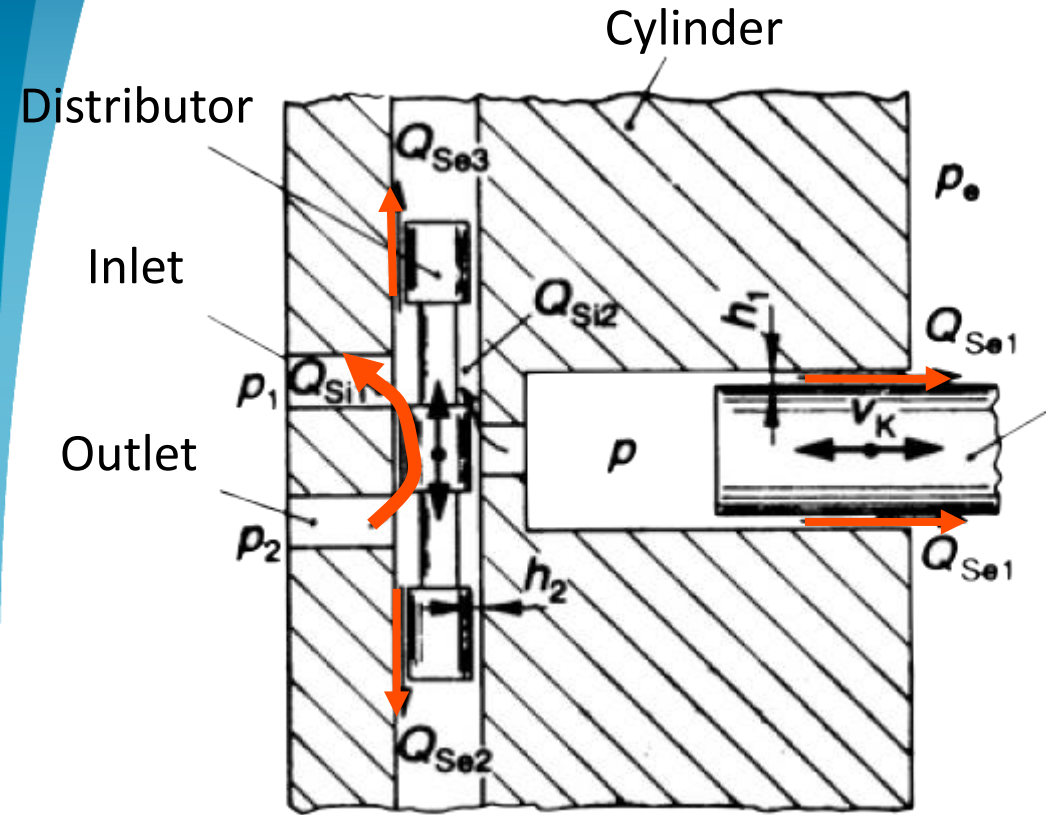
B C F G I

Torque Losses

A B C D E

G H J

# Πραγματική μηχανή θετικής μετατόπισης



$Q_{Se} \dots$  external volumetric losses

$Q_{Si} \dots$  internal volumetric losses

Effective Flow rate:

$$Q_e = \alpha V_{max} n - Q_s$$

$Q_s \dots$  volumetric losses

Effective torque:

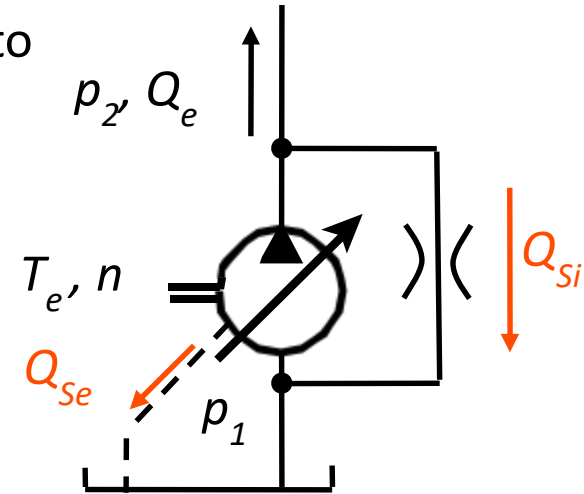
$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_s$$

$T_s \dots$  torque losses

# Ογκομετρικές απώλειες

$$Q_S = \sum_{i=1}^n Q_{Sei} + \sum_{j=1}^m Q_{Sij} + Q_{SK} + Q_{Sf}$$

$\uparrow$  external volumetric losses  
 $\uparrow$  internal  
 $\uparrow$  losses due to compressibility  
 $\uparrow$  losses due to Incomplete filling

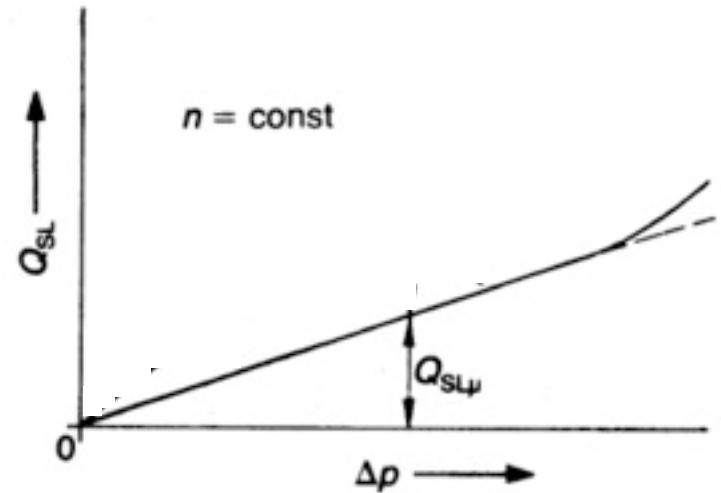


$Q_{SL}$  external and internal volumetric losses = flow through laminar resistances:

$$Q_{SL} = C_{\mu} \cdot \frac{\Delta p}{\mu}$$

$$Q = \frac{b \cdot h^3 \cdot \Delta p}{12 \cdot \mu}$$

Assuming const. gap height



Dynamic viscosity

$$\mu = f(\theta, p)$$

# Στιγμιαία παροχή αντλίας

Instantaneous volumetric flow  $Q_a$

$$Q_a = \frac{dV}{dt} = f(\varphi)$$

Volumetric flow displaced by  
a displacement chamber

$$Q_{ai} = f(\varphi_i)$$

The instantaneous volumetric flow is given by the sum of instantaneous flows  $Q_{ai}$  of each displacement element:

$$Q_a = \sum_{i=1}^k Q_{ai}$$

$k$  ... number of displacement chambers, decreasing  
their volume, i.e. being in the delivery stroke

$z$  is an **even** number

$$k = \frac{z}{2}$$

$z$  ... number of displacement elements

$z$  is an **odd** number

$$k = \frac{z}{2} + 0.5 \quad \text{or} \quad k = \frac{z}{2} - 0.5$$



Flow pulsation of pumps



Pressure pulsation

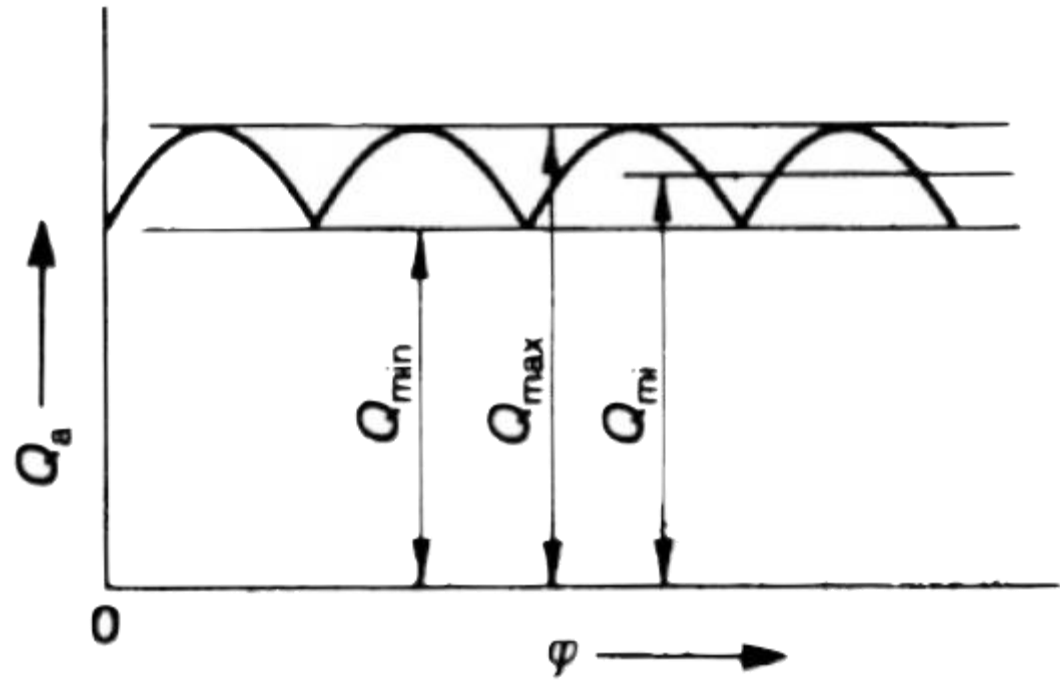
# Παλμός παροχής

Non-uniformity grade of volumetric flow is defined:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}}$$

$$Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$

$$\delta_Q = 2 \frac{Q_{\max} - Q_{\min}}{Q_{\max} + Q_{\min}}$$



## Απώλειες ροπής

$$T_S = T_{S\mu} + T_{Sp} + T_{Sp} + T_{Sc} \Rightarrow \text{constant value}$$

Torque loss due to viscous friction in gaps (laminar flow)

$$T_{S\mu} = k_{T\mu} \cdot \frac{\mu}{h} \cdot n = C_{T\mu} \cdot \mu \cdot n$$

*h...gap height*

Torque loss to overcome pressure drop caused in turbulent resistances

$$T_{Sp} = C_{Tp} \rho n^2$$

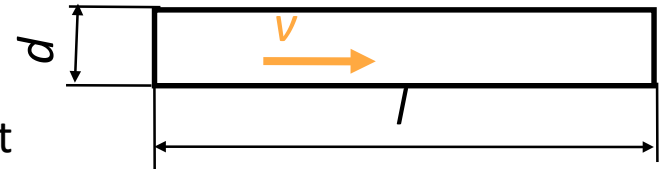
$$\Delta p_s = \lambda \cdot \frac{l}{d} \cdot \rho \cdot \frac{v^2}{2} + \xi \cdot \rho \cdot \frac{v^2}{2}$$

Torque loss linear dependent on pressure

$$T_{Sp} = C_{Tp} \Delta p$$

$\xi$  ... drag coefficient

$\lambda$  ... flow resistance coefficient



$$\lambda_{turbulent} = \frac{0.3164}{\sqrt[4]{Re}}$$



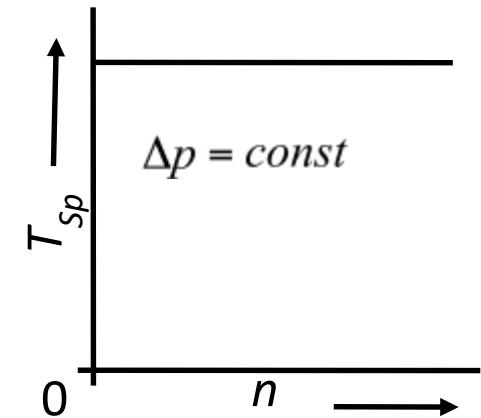
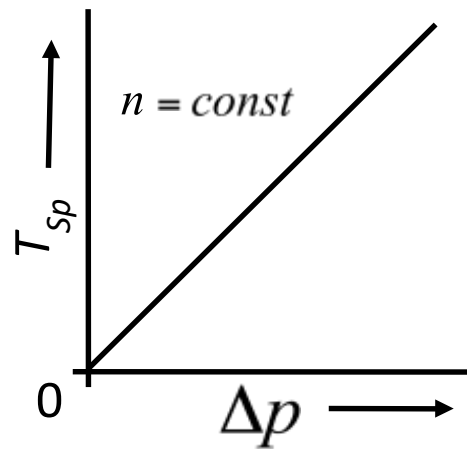
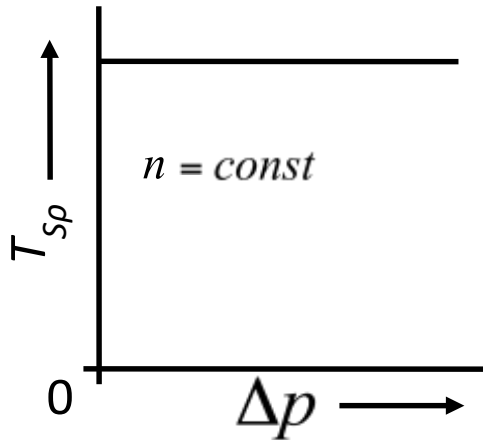
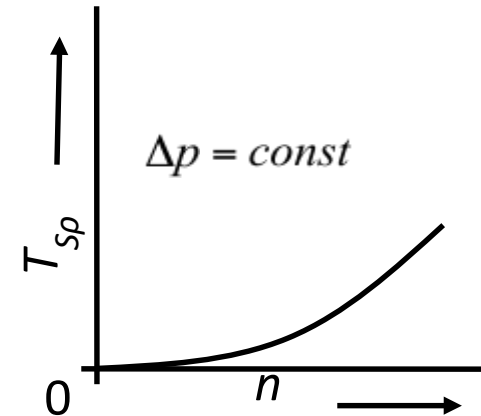
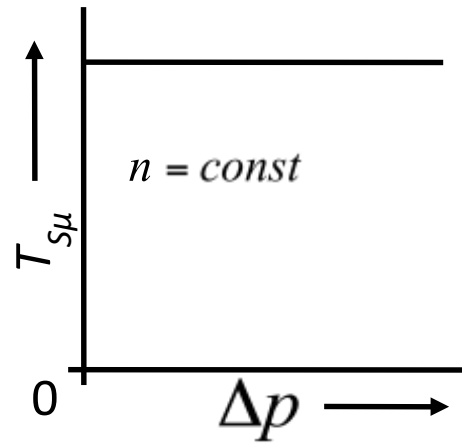
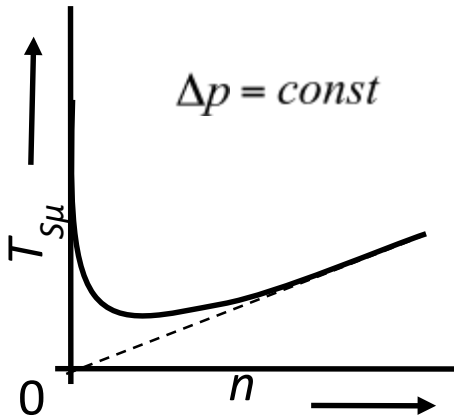
$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{\max}}{2 \cdot \pi} + T_S$$

effective torque required at pump shaft

# Συμπεριφορά steady state

Torque losses of  
a real displacement machine

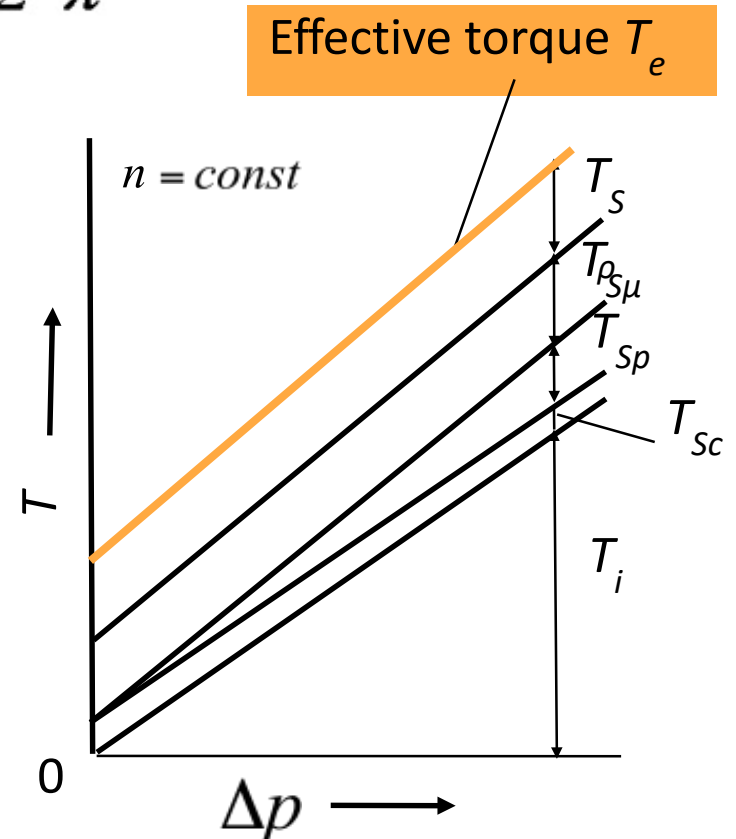
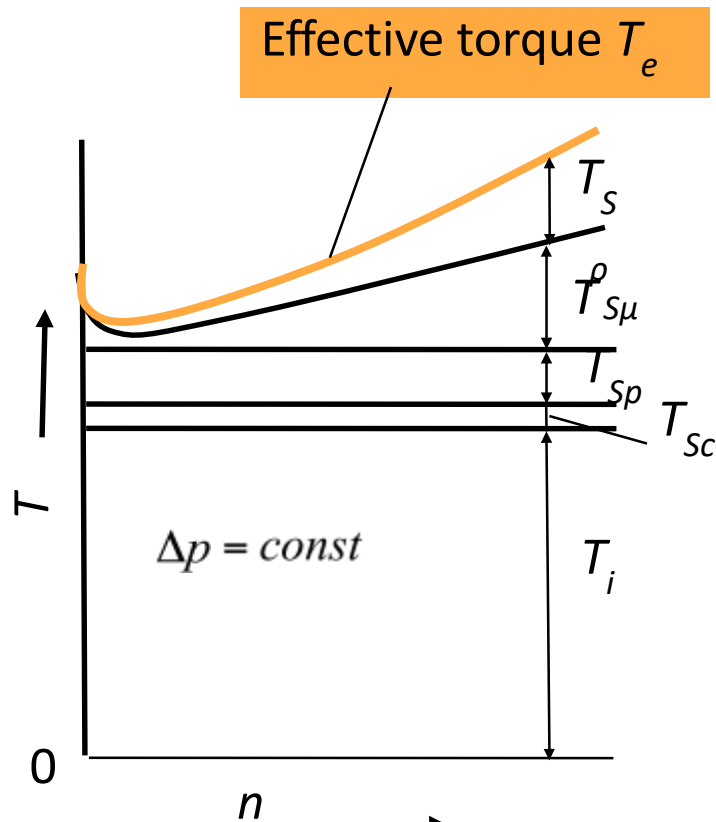
$$T_S = f(n, \Delta p, V, \theta)$$





# Συμπεριφορά steady state

$$T_e = T_i + T_s = \frac{\Delta p \cdot \alpha \cdot V_{\max}}{2 \cdot \pi} + T_s$$



→  $T_s = f(\Delta p, n, V, \theta)$

# Αντλία αξονικών εμβόλων-Κινηματικά

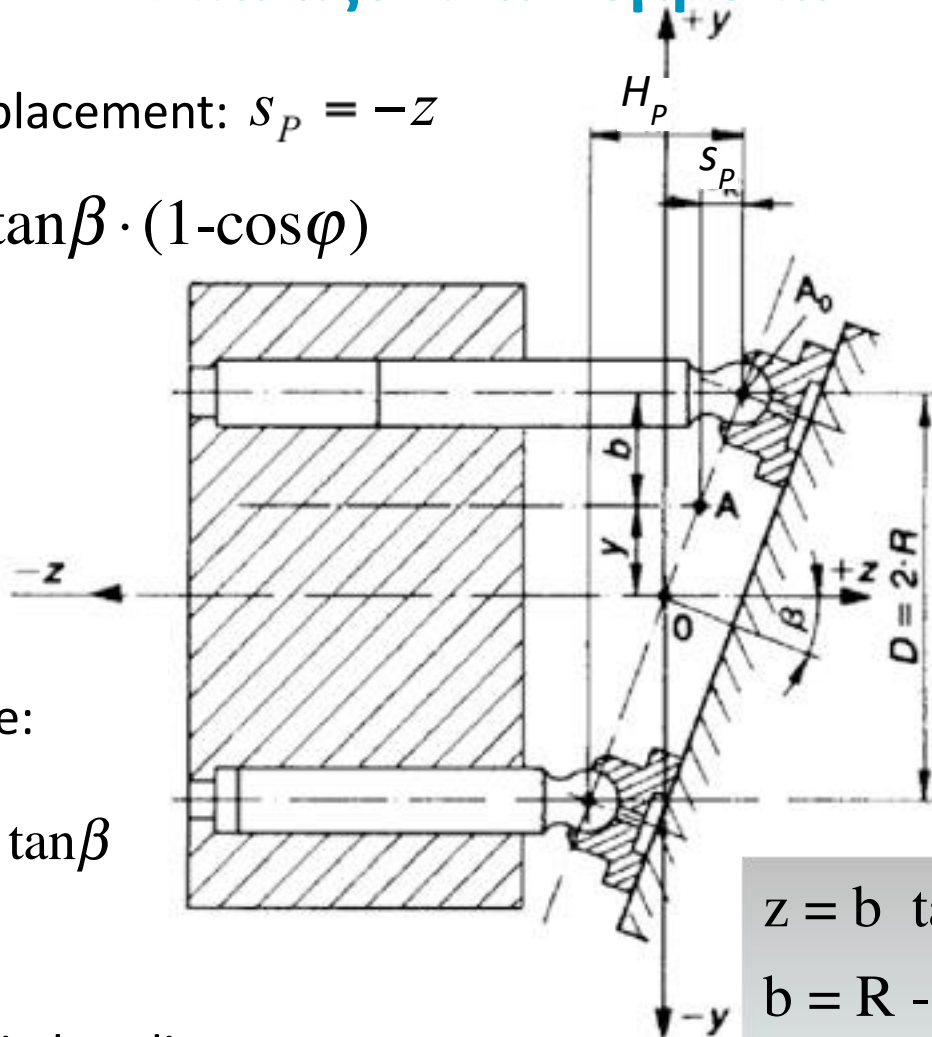
Piston displacement:  $s_p = -z$

$$s_p = -R \cdot \tan\beta \cdot (1 - \cos\varphi)$$

Piston stroke:

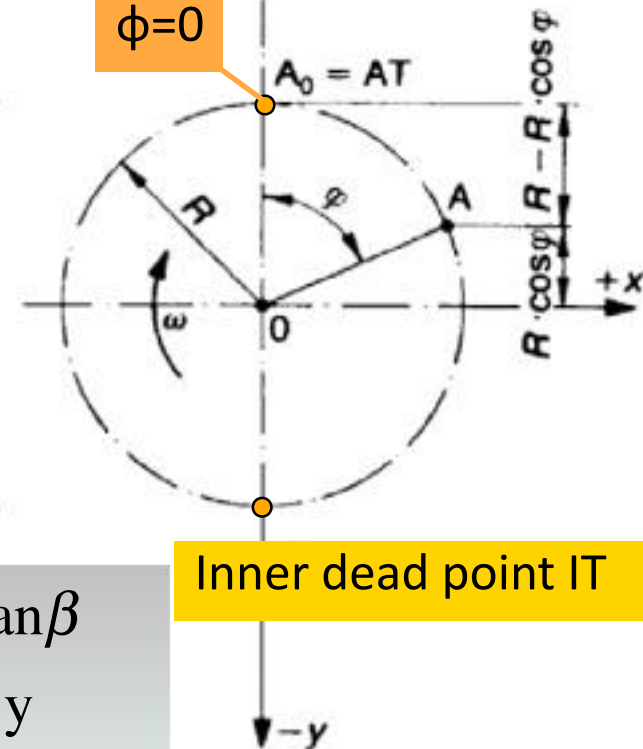
$$H_p = 2 \cdot R \cdot \tan\beta$$

R ... pitch radius



Outer dead point AT

$$\phi = 0$$



Inner dead point IT

$$z = b \tan\beta$$

$$b = R - y$$

$$y = R \cdot \cos\varphi$$

# Αντλία αξονικών εμβόλων-Κινηματικά

Piston velocity in z-direction:

$$v_P = \frac{ds_P}{dt} = \frac{ds_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega \cdot R \cdot \tan \beta \cdot \sin \varphi$$

Piston acceleration in z-direction:

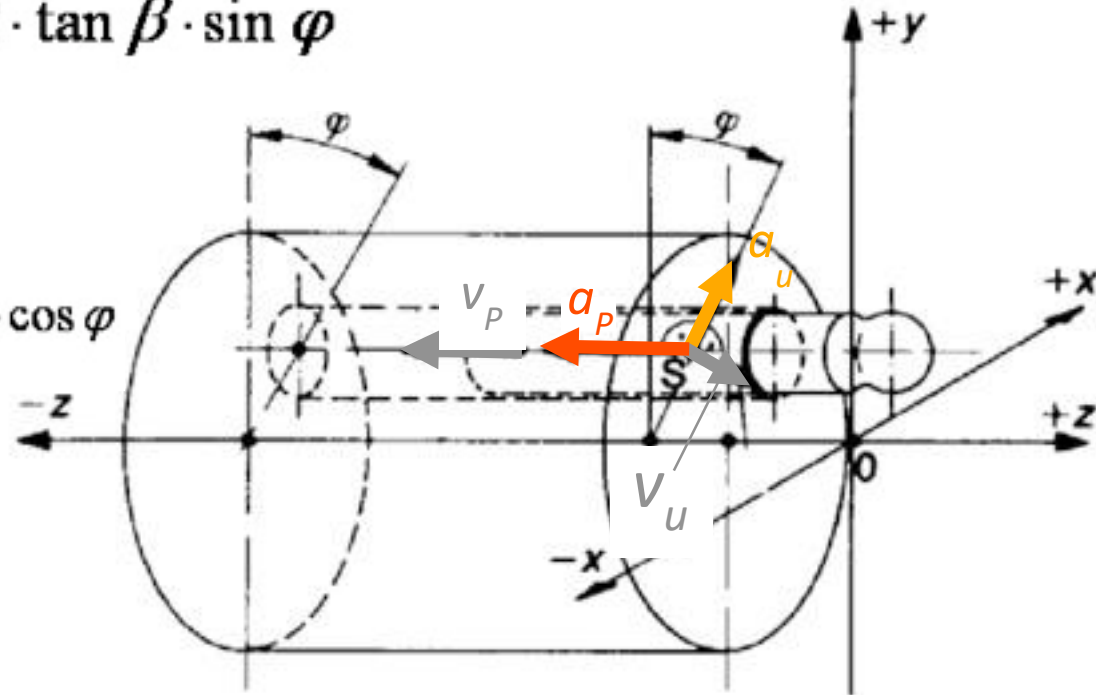
$$a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega^2 \cdot R \cdot \tan \beta \cdot \cos \varphi$$

Circumferential speed

$$v_u = R \omega$$

Centrifugal acceleration:

$$a_u = R \omega^2$$



Coriolis acceleration  $a_c$  is just zero, as the vector of angular velocity  $\omega$  and the piston velocity  $v_p$  run parallel

# Στιγμαία παροχή

Geometric displacement volume:

$$V_g = z \cdot A_p \cdot H_p$$

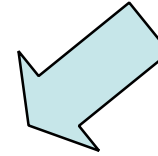
$z$  ... number of pistons

In case of pistons arranged parallel to shaft axis:

$$V_g = z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta$$

Geometric flow rate:  $Q_g = n \cdot z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta$

For an ideal pump without losses



Mean value over time

Instantaneous volumetric flow:

$$Q_a = \sum_{i=1}^k Q_{ai}$$

$k$  ... number of pistons, which are in the delivery stroke

with  $Q_{ai}$  instantaneous volumetric flow of individual piston

$$Q_{ai} = f(\varphi_i)$$

$$v_p = \omega \cdot R \cdot \tan \beta \cdot \sin \varphi$$

$$Q_{ai} = v_p \cdot A_p = \omega \cdot A_p \cdot R \cdot \tan \beta \cdot \sin \varphi_i$$

# Στιγμαία παροχή

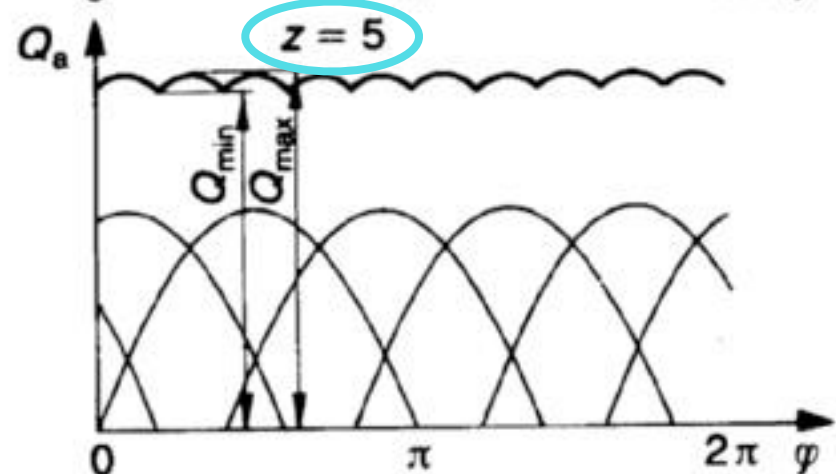
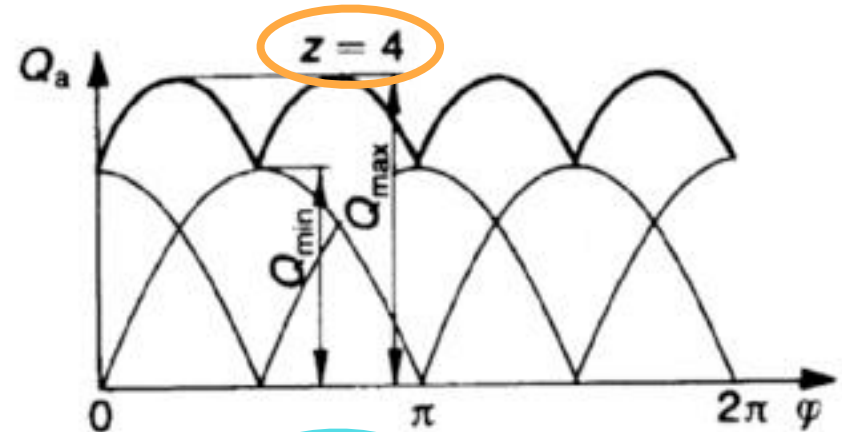
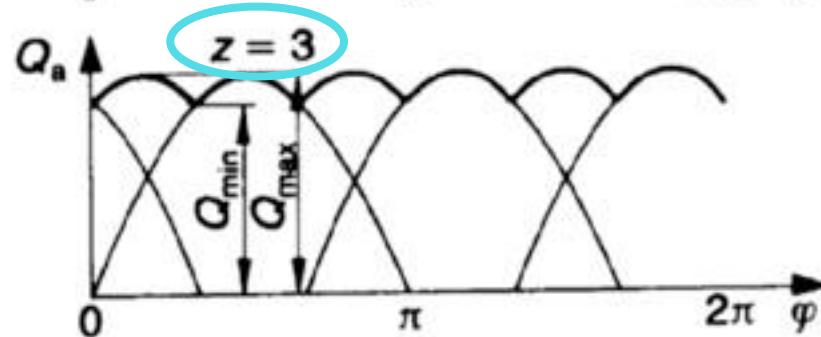
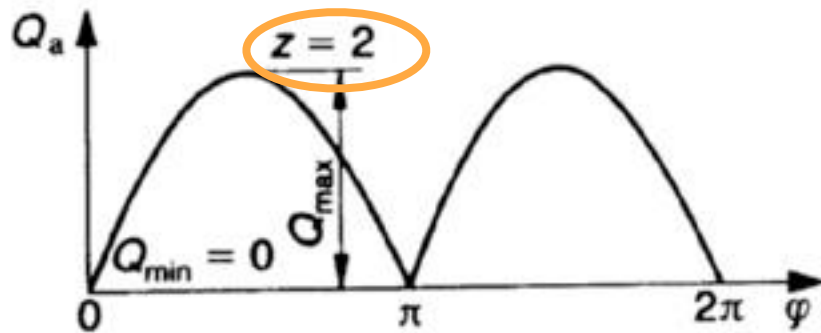
In case of **even** number of pistons:  $k = 0.5 \cdot z$

In case of **odd** number of pistons:

$$k_1 = \frac{z}{2} + 0.5 \quad \text{for } 0 < \varphi \leq \frac{\pi}{z}$$

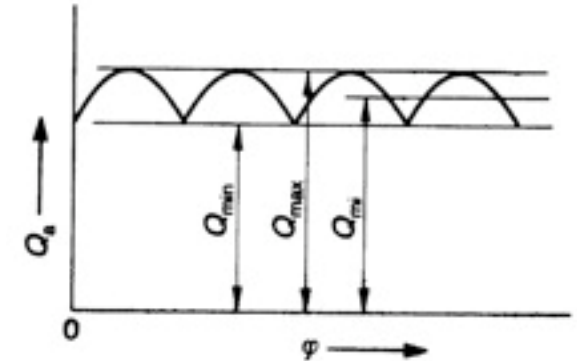
$$\text{and } k_2 = \frac{z}{2} - 0.5 \quad \text{for } \frac{\pi}{z} < \varphi \leq 2 \cdot \frac{\pi}{z}$$

$$Q_a = \sum_{i=1}^k Q_{ai}$$



## Παλμός παροχής και ροπής

kinematic flow and torque pulsation due to a finite number of piston



Flow Pulsation:

Non-uniformity grade:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$

Even number of pistons:

Odd number of pistons:

$$\delta_Q = \frac{\pi}{z} \tan \frac{\pi}{2z}$$

$$\delta_Q = \frac{\pi}{2z} \tan \frac{\pi}{4z}$$

Torque Pulsation

$$\delta_T = \frac{T_{\max} - T_{\min}}{T_{mi}} \quad \text{with} \quad T_{mi} = \frac{T_{\max} + T_{\min}}{2}$$

## Παλμός παροχής και ροπής

kinematic flow and torque pulsation due to

a finite number of piston

z... number of pistons

Non-uniformity

Even number of pistons:

Odd number of pistons:

$$\delta_Q = \delta_T = \frac{Q_{\max} - Q_{\min}}{Q_{\text{mean}}} = \frac{T_{\max} - T_{\min}}{T_{\text{mean}}}$$

$$\delta_Q = \frac{\pi}{z} \cdot \tan \frac{\pi}{2 \cdot z}$$

$$\delta_Q = \frac{\pi}{2 \cdot z} \cdot \tan \frac{\pi}{4 \cdot z}$$

NUMBER OF PISTONS	3	4	5	6	7	8	9	10	11	12	13	14
NON-UNIFORMITY of FLOW / TORQUE	0,1403	0,3253	0,0498	0,1403	0,0253	0,0781	0,0153	0,0498	0,0102	0,0345	0,0073	0,0253

NUMBER OF PISTONS	15	16	17	18	19	20	21	22	23	24	25	26
NON-UNIFORMITY of FLOW / TORQUE	0,0055	0,0193	0,0043	0,0153	0,0034	0,0124	0,0028	0,0102	0,0023	0,0086	0,0020	0,0073

Flow and torque pulsation frequency f:

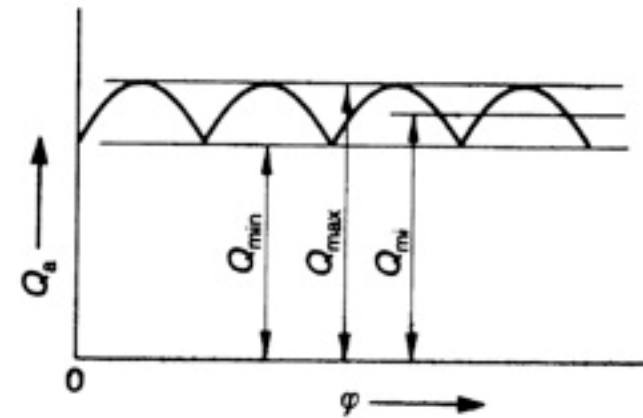
Even number of pistons:  $f = z \cdot n$

Odd number of pistons:  $f = 2 \cdot z \cdot n$

# Παλμός παροχής

Non-uniformity grade:

$$\delta_Q = \frac{Q_{\max} - Q_{\min}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\max} + Q_{\min}}{2}$$



Kinematic non-uniformity grade for piston machines:

Number of pistons z	3	4	5	6	7	8	9	10	11
Non-uniformity grade $\delta$	0.140	0.325	0.049	0.140	0.025	0.078	0.015	0.049	0.010



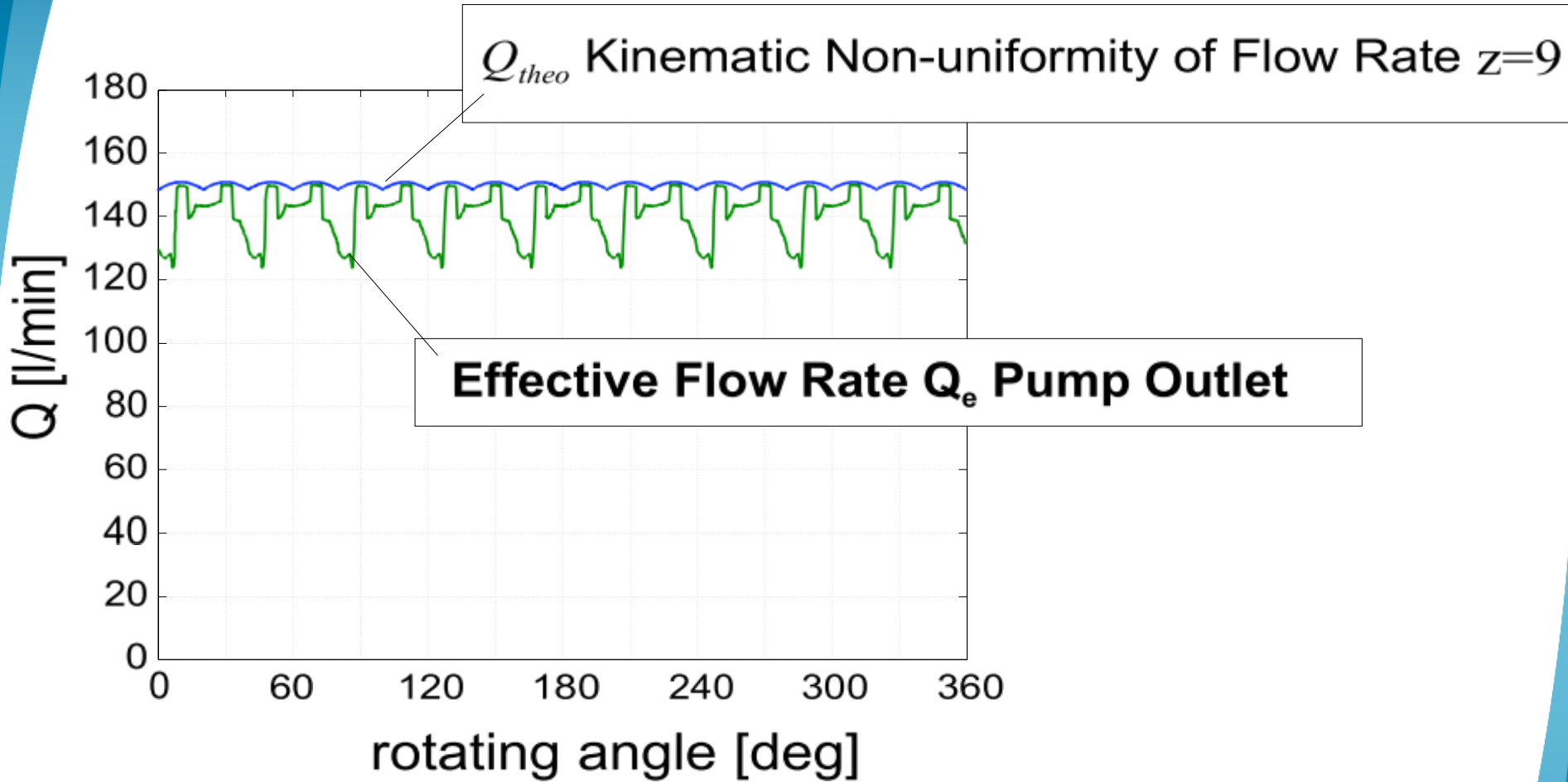
Volumetric losses  $Q_s = f(\phi)$  and

$$Q_s = f(\Delta p, n, V_i, \theta)$$

Flow pulsation of a real displacement machine is much larger than the flow pulsation given by the kinematics



# Παλμός παροχής



Flow pulsation leads to pressure pulsation at pump outlet

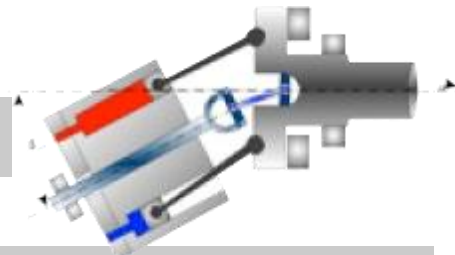
# Μηχανές θετικής μετατόπισης

Piston Machines

Axial Piston Machines



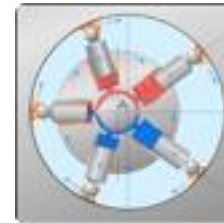
Swash Plate Machines



Bent Axis machines

In-line Piston Machines

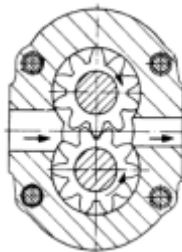
Radial Piston Machines



with external piston support

with internal piston support

Gear Machines

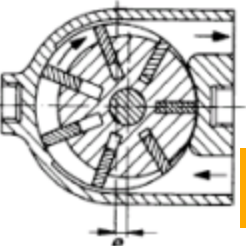
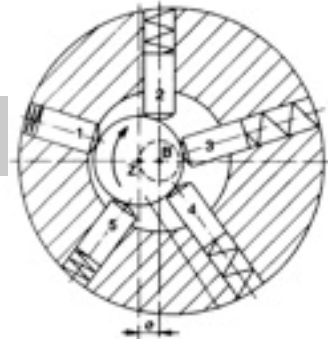


External Gear

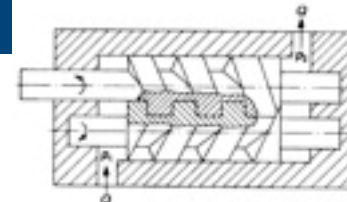
Internal Gear



Annual Gear



Vane Machines



Screw Machines

others

Fixed displacement machines



Variable displacement machines

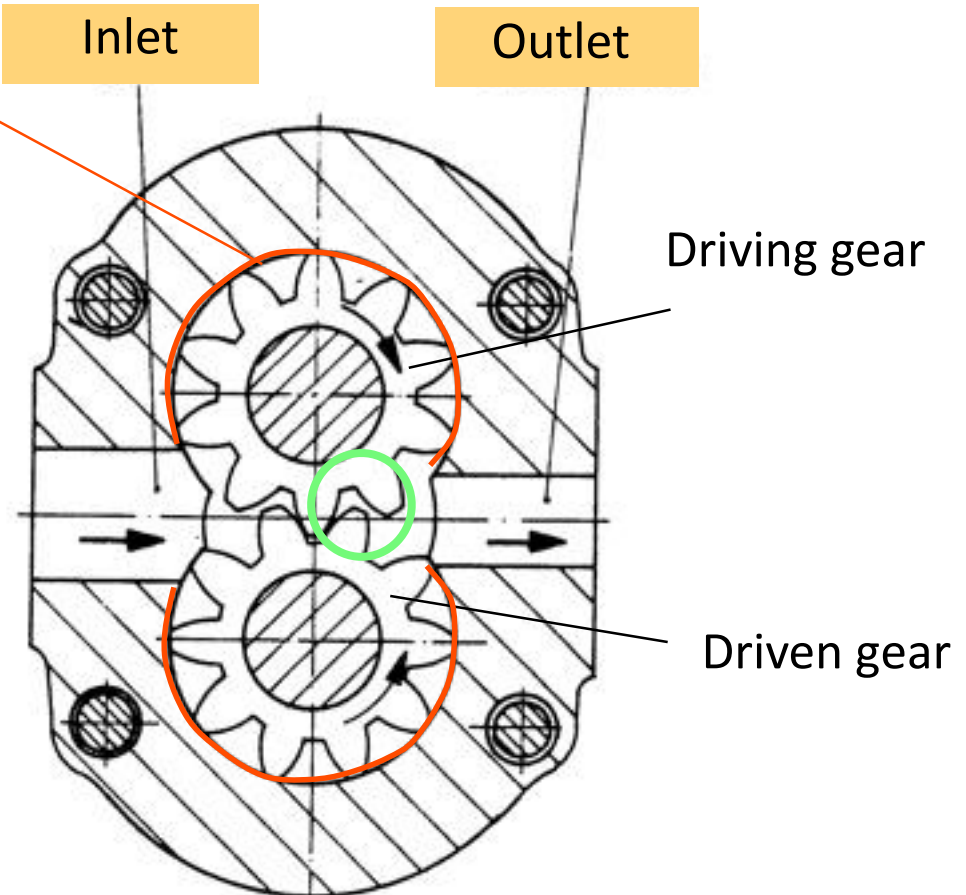
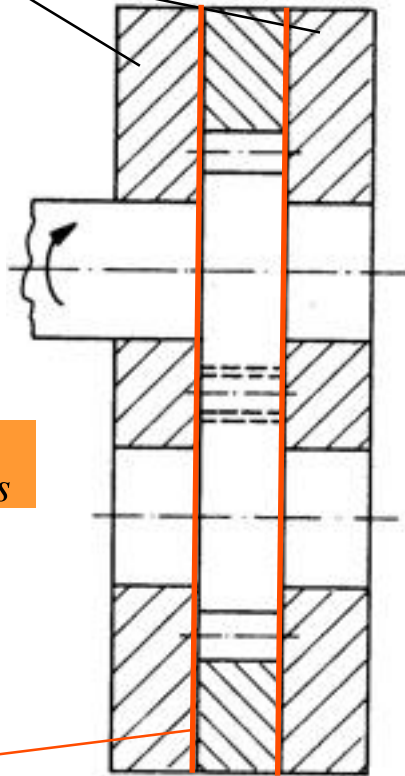
# Γραναζωτή αντλία εξωτερικής οδόντωσης

Radial gaps between teeth addendum circle and housing

Housing

Inlet

Outlet



$$Q_e = V \cdot n - Q_s$$

Axial gaps between housing and the gear pair must be very small to seal the displacement chamber

# Expression for the theoretical flow rate of an external gear pump

$D_o$  = the outside diameter of gear teeth

$D_i$  = the inside diameter of gear teeth

$L$  = the width of gear teeth

$N$  = the speed of pump in RPM

$V_D$  = the displacement of pump in m<sup>3</sup>/rev

$M$  = module of gear

$z$  = number of gear teeth

$\alpha$  = pressure angle

Volume displacement is

$$V_D = \frac{\pi}{4} (D_o^2 - D_i^2) L$$

$$D_i = D_o - 2(\text{Addendum} + \text{Dendendum})$$

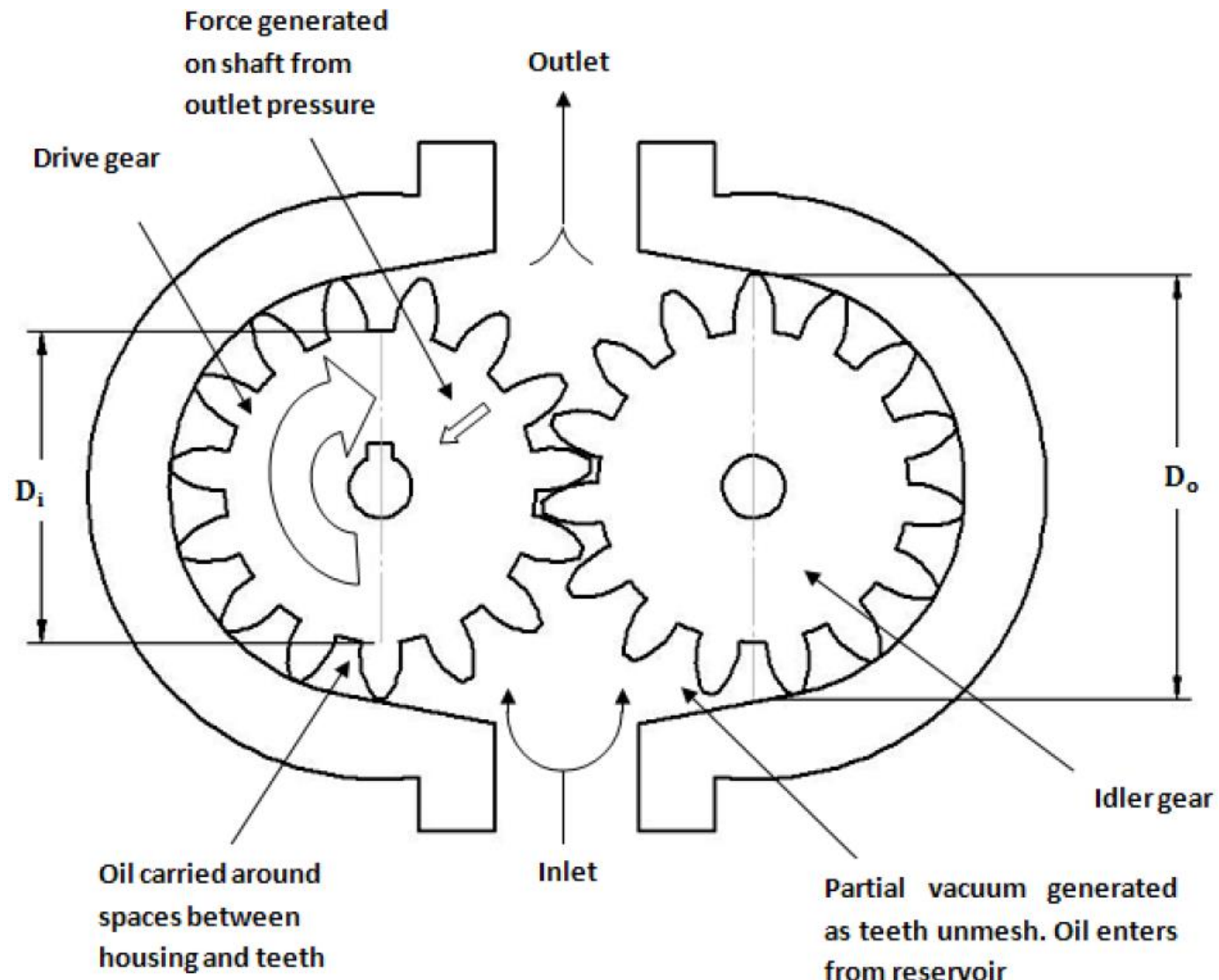
Theoretical discharge is

$$Q_T \text{ (m}^3\text{/min)} = V_D \text{ (m}^3\text{/rev)} \times N \text{ (rev/min)}$$

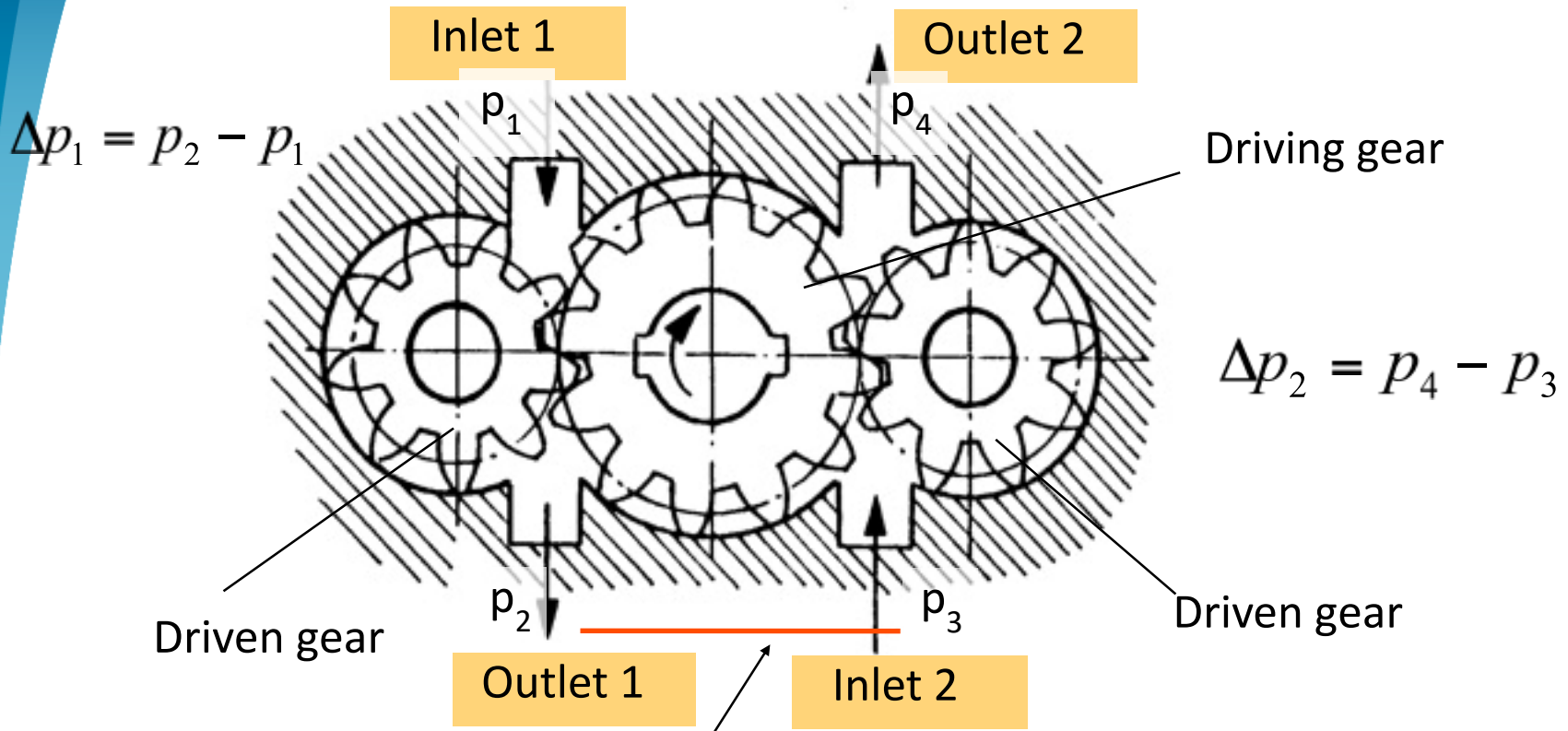
If the gear is specified by its module and number of teeth, then the theoretical discharge can be found by

$$Q_T = 2\pi L m^2 N \left[ z + \left( 1 + \frac{\pi^2 \cos^2 20}{12} \right) \right] \text{ m}^3\text{/min}$$

# Expression for the theoretical flow rate of an external gear pump



# Γραναζωτή αντλία εξωτερικής οδόντωσης δυο βαθμίδων



Outlet 1 and inlet 2 can be connected



$$p_2 = p_3$$

or the pump can have two separate outlets



$$p_1 = p_3$$

$$\Delta p_1 \approx \Delta p_2$$



the driving gear is pressure balanced!

# Γραναζωτή αντλία εσωτερικής οδόντωσης

Internal gear (ring gear)

Pinion

Advantages:

Better suction ability

Higher efficiency

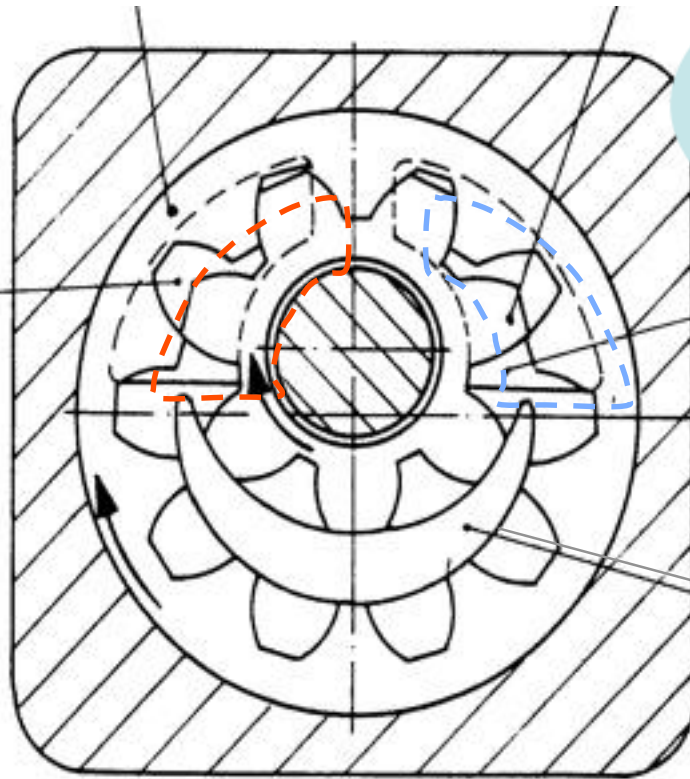
More compact design

Less noise emission

Pressure zone

Suction zone

Crescent shaped divider



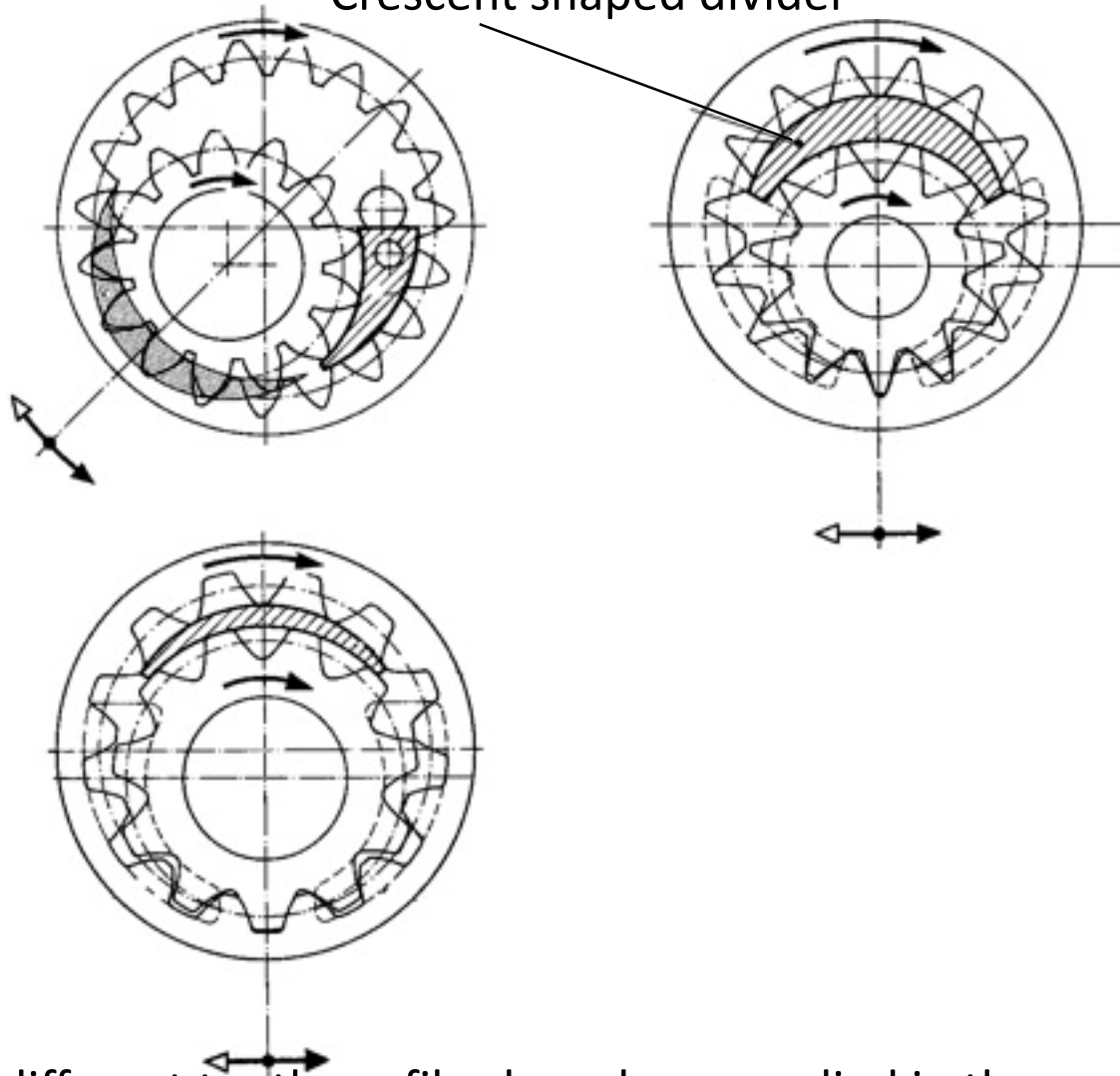
Using teeth of standard involute design requires a combination where the pinion has two or more fewer teeth than the ring gear! Pinion and ring gear are then separated by a crescent shaped divider.



Longer duration of teeth meshing leads to better sealing function

# Γραναζωτή αντλία εσωτερικής οδόντωσης

Crescent shaped divider

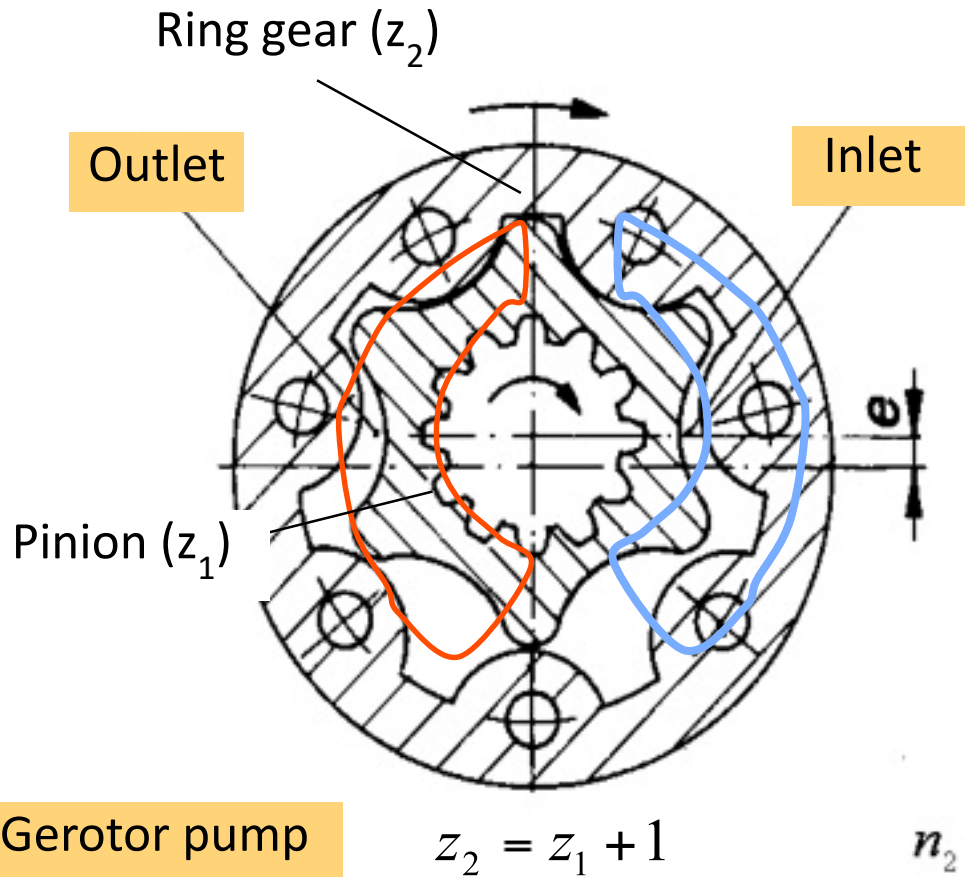


Many different tooth profiles have been applied in the recent past.



# Αντλίες τύπου gerotor

Applying specially generated tooth curves it can be achieved, that the inner rotor (the pinion) has only one tooth less than the ring gear, thus eliminating the crescent-shaped divider.



Each tooth of the pinion maintains continuous sliding contact with a tooth of the ring gear, providing fluid tight engagement.

Relative sliding velocity between pinion and ring gear is very small

quiet operation and long service life

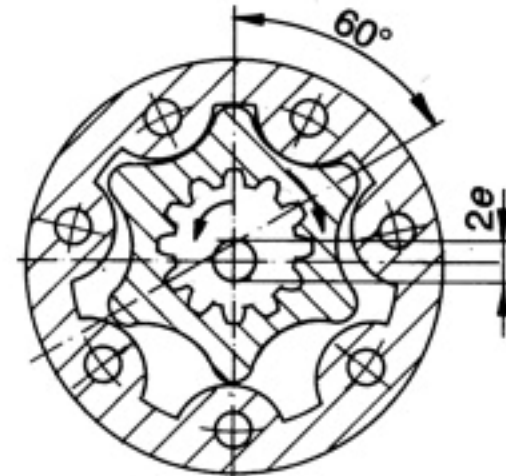
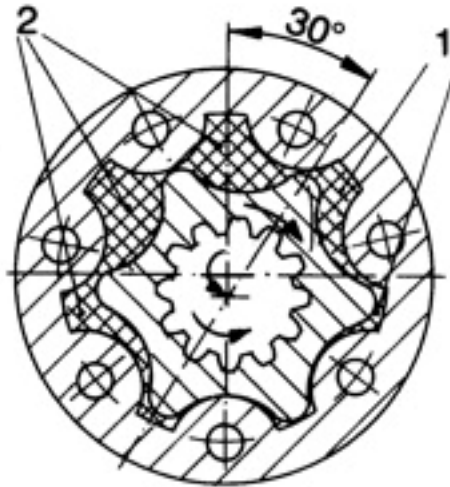
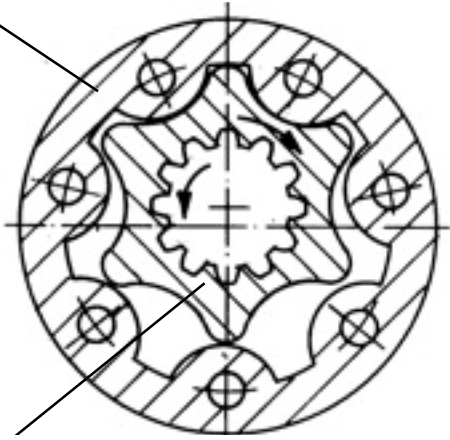
$$n_2 = n_1 \cdot \frac{z_1}{z_2} = n_1 \cdot \left(1 - \frac{1}{z_2}\right) = n_1 \cdot \left(1 + \frac{1}{z_1}\right)^{-1}$$

# Αντλίες τύπου Gerotor

2 Pressure port

$$z_2 = z_1 + 1$$

Ring gear ( $z_2$ ) fixed



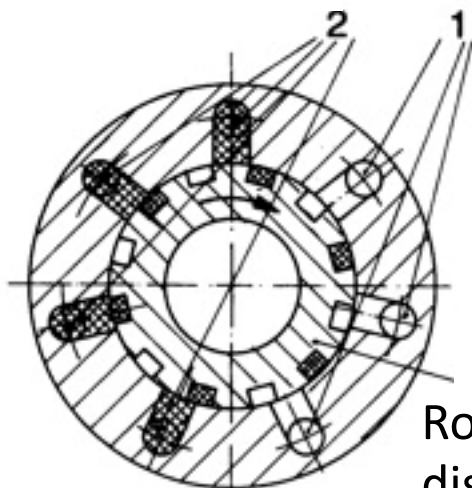
Rotating pinion ( $z_1$ )

Outlet

Inlet

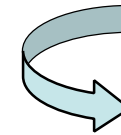
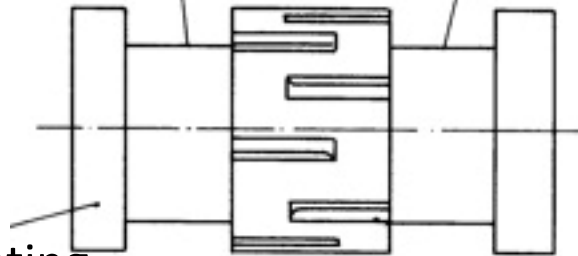
Displacement volume is given by  $z_1$  times  $z_2$  tooth spaces

Multiple delivery of each tooth space



Rotating distributor

1 Suction port



# Αξονικά διάκενα

Sealing ring

Gear pair

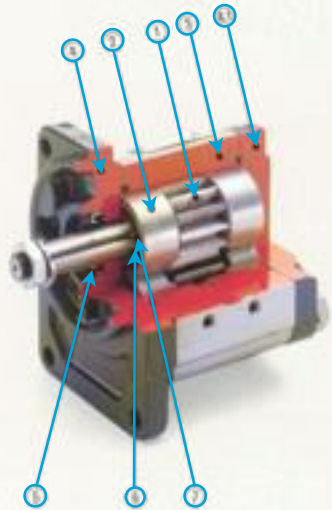
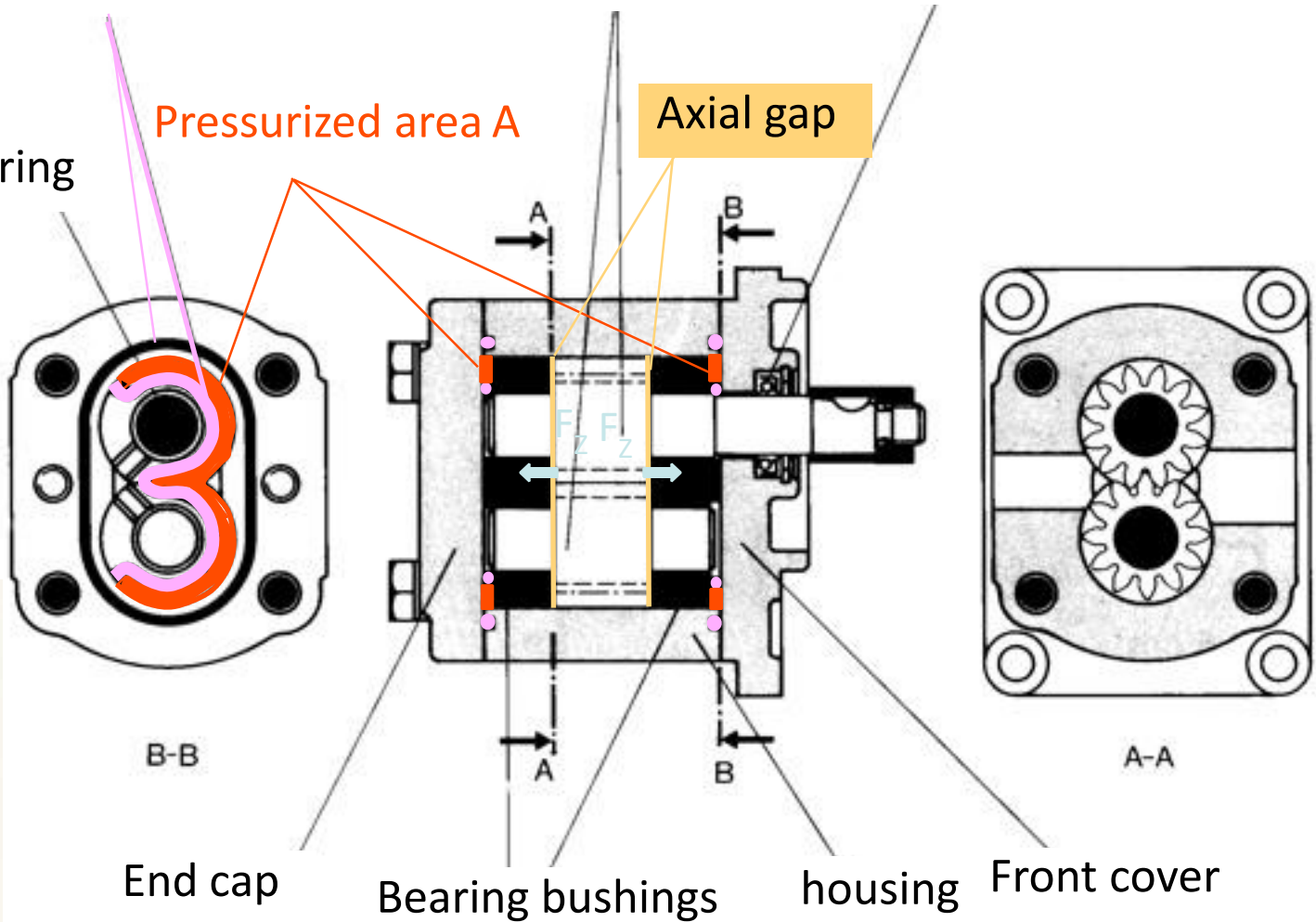
Shaft seal

$$A = (1.1 \div 1.3) \cdot \frac{F_z}{p_2}$$

Sliding bearing

Pressurized area A

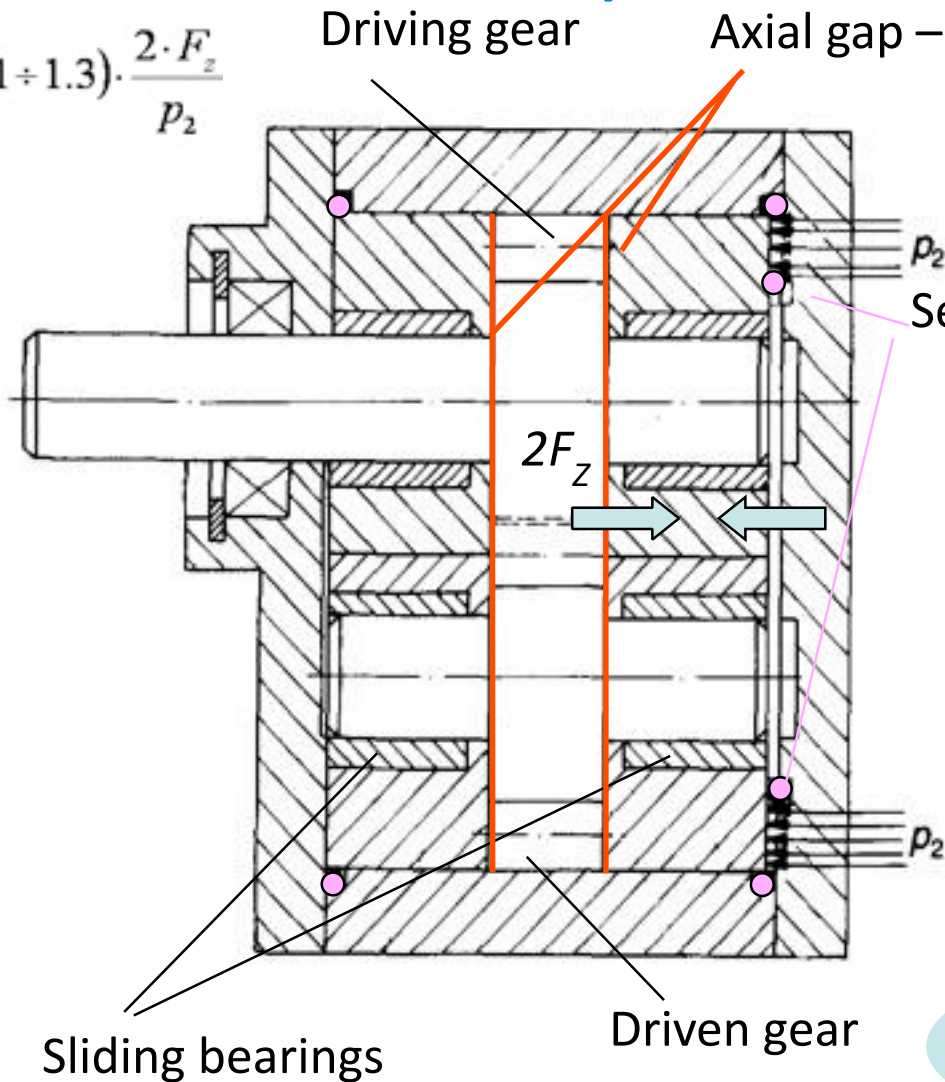
Axial gap



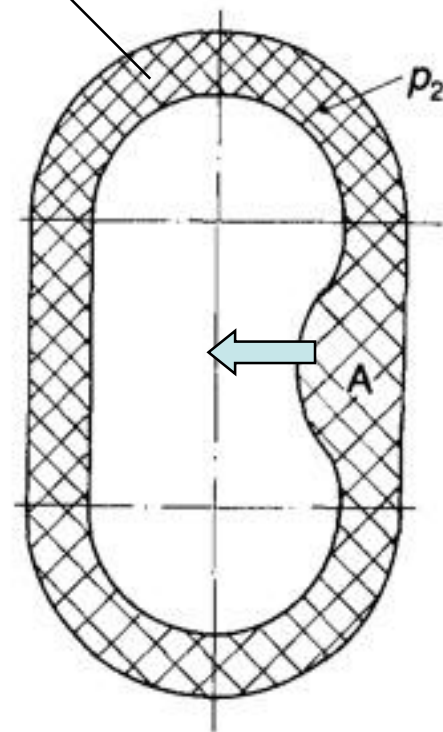
Only one direction of shaft rotation possible!

# Αξονικά διάκενα

$$A = (1.1 \div 1.3) \cdot \frac{2 \cdot F_z}{p_2}$$



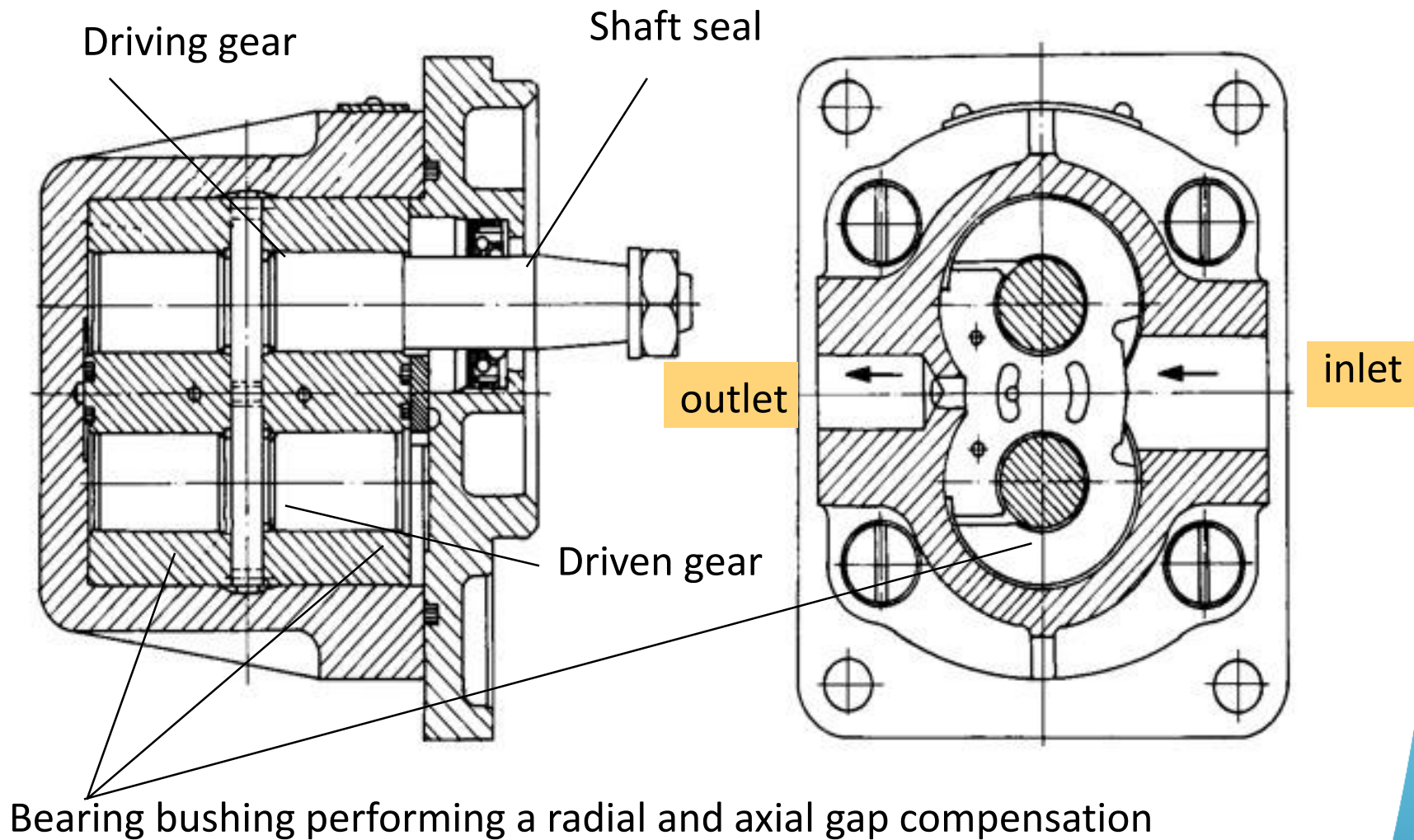
Pressurized area



Improved volumetric efficiency

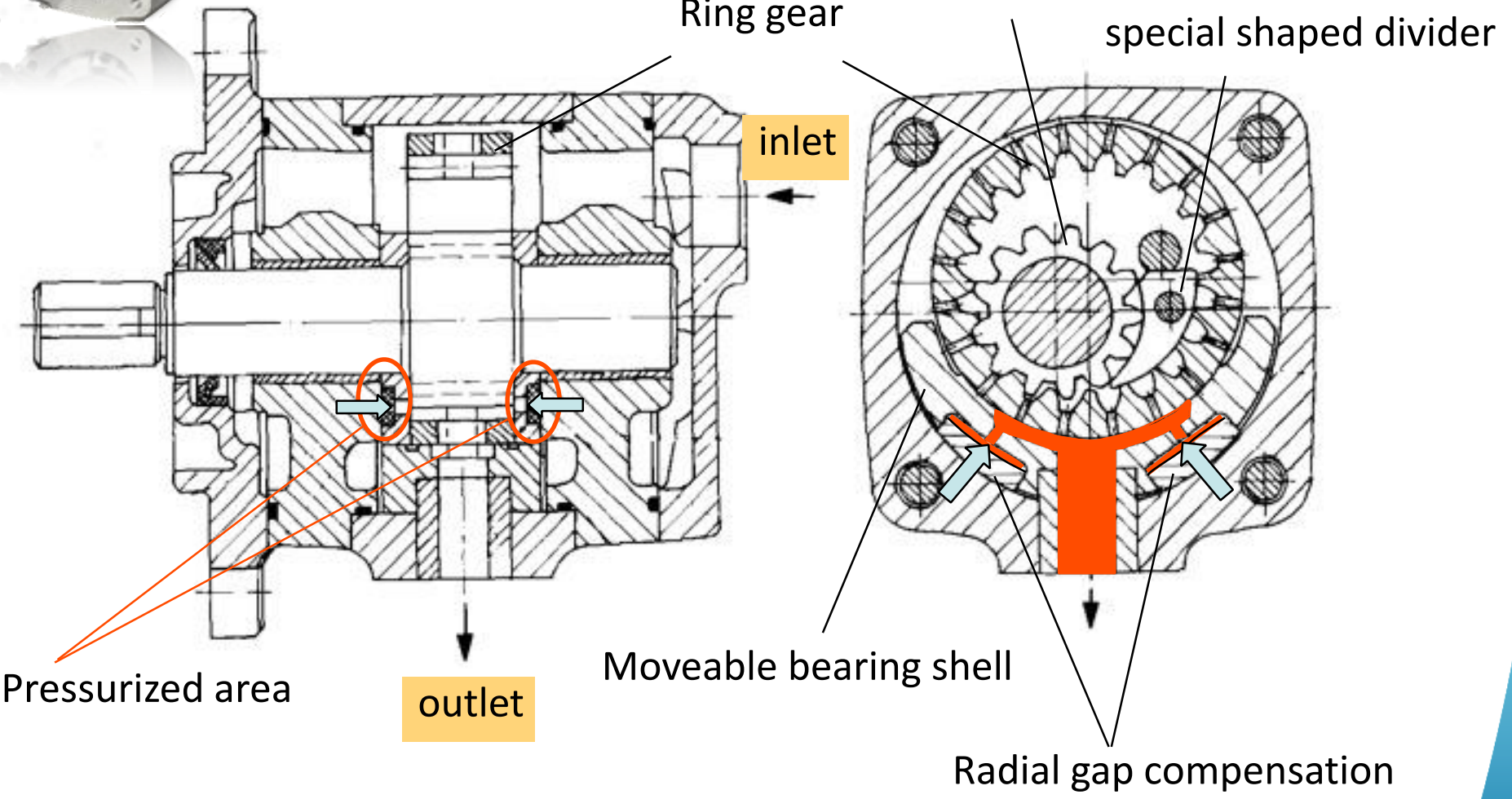


# Παράδειγμα σχεδιασμού



# Παράδειγμα σχεδιασμού

Internal gear pump with axial and radial gap compensation



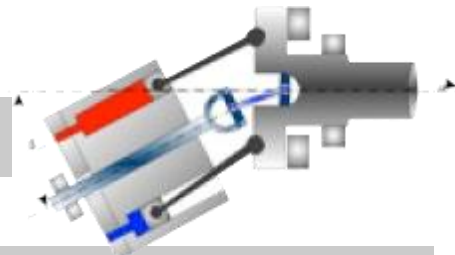
# Μηχανές θετικής μετατόπισης

Piston Machines

Axial Piston Machines



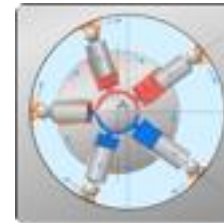
Swash Plate Machines



Bent Axis machines

In-line Piston Machines

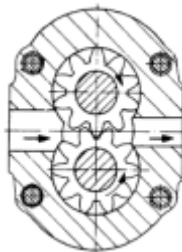
Radial Piston Machines



with external piston support

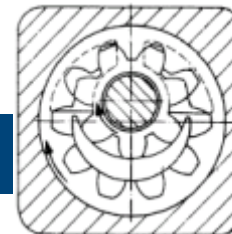
with internal piston support

Gear Machines

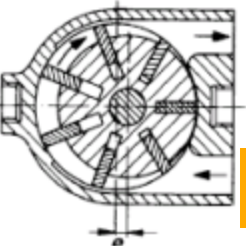
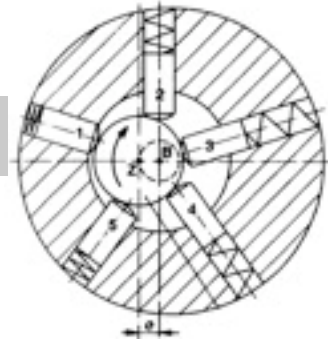


External Gear

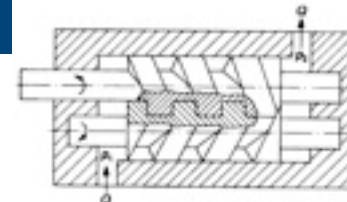
Internal Gear



Annual Gear



Vane Machines



Screw Machines

others

Fixed displacement machines

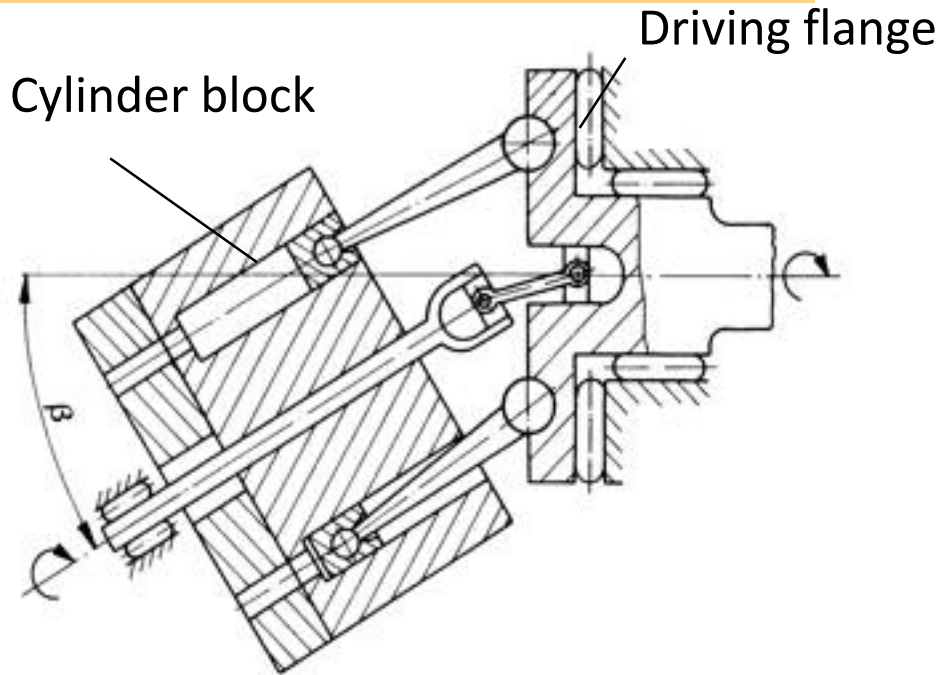


Variable displacement machines



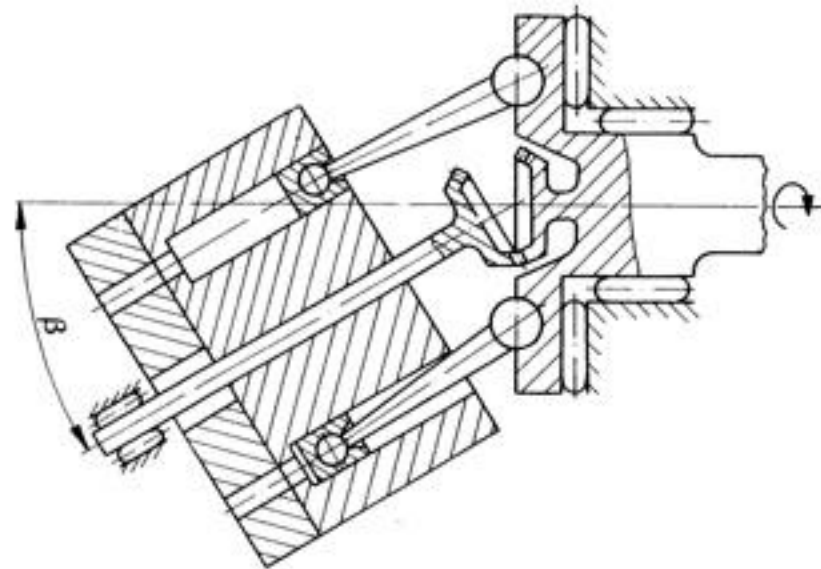
# Αντλίες αξονικών εμβόλων bent axis

## Synchronization of cylinder block



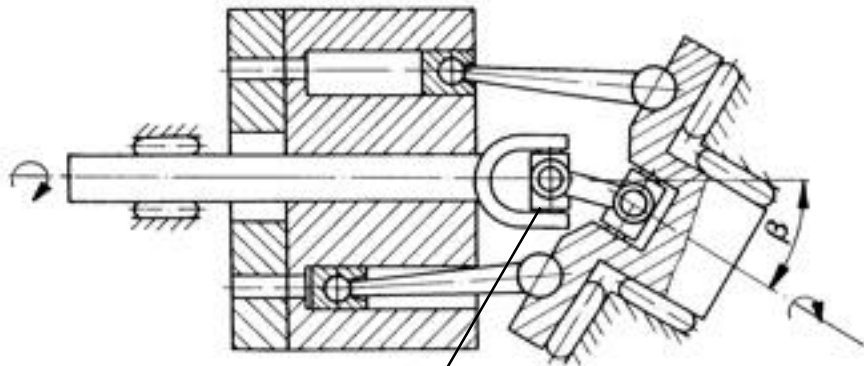
Using a universal joint

Using a bevel gear

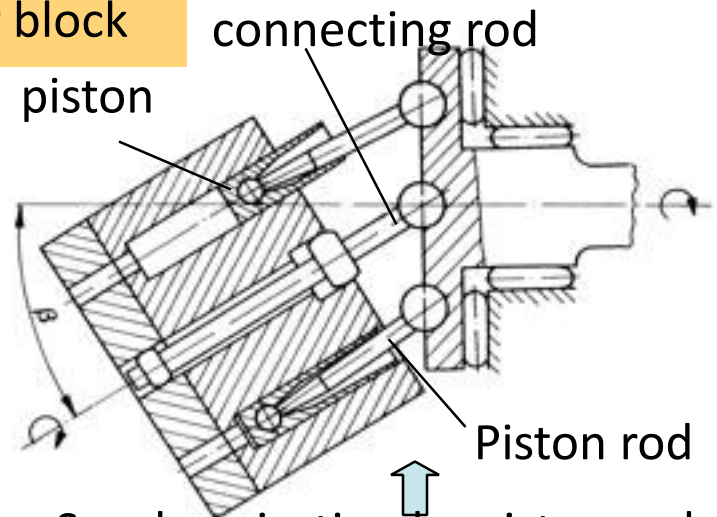


# Αντλίες αξονικών εμβόλων bent axis

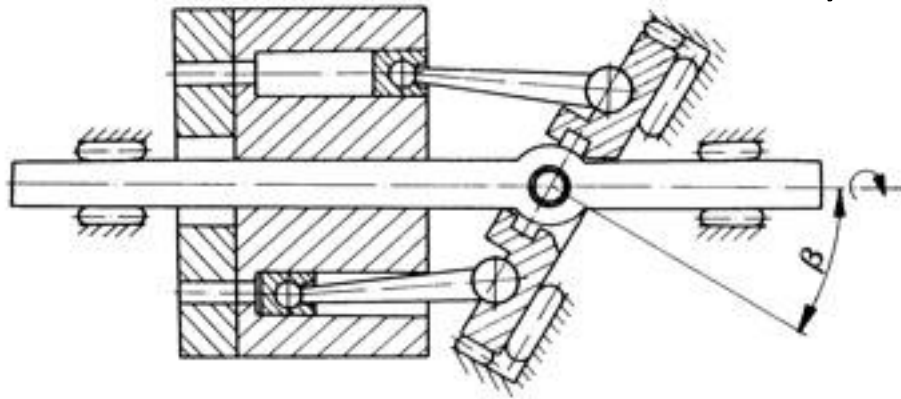
Synchronization of cylinder block



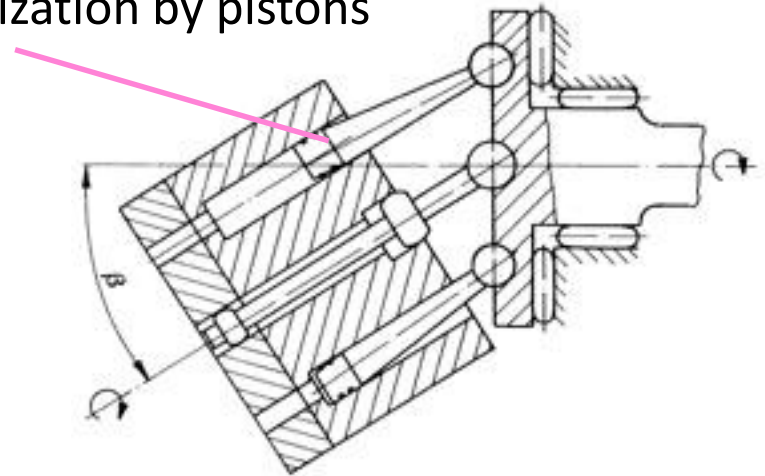
Cardan joint



Synchronization by piston rod

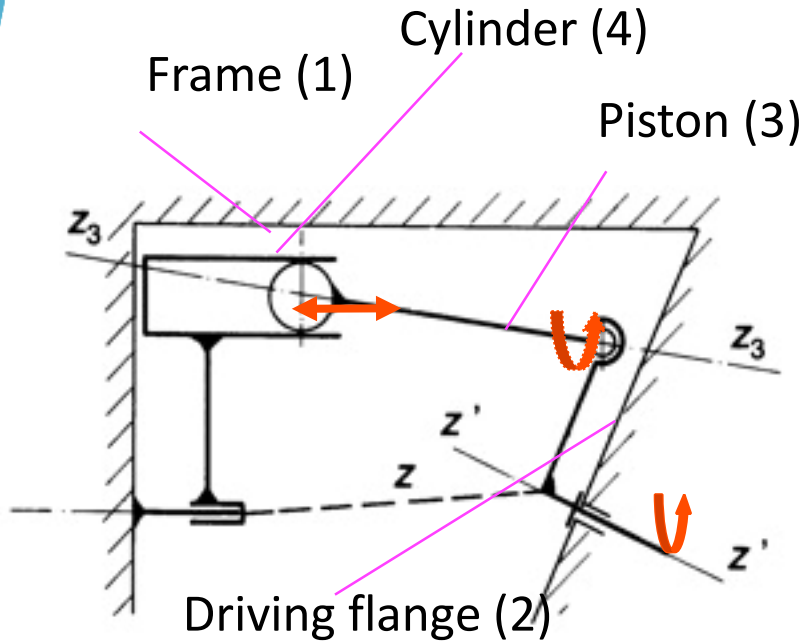


Synchronization by pistons

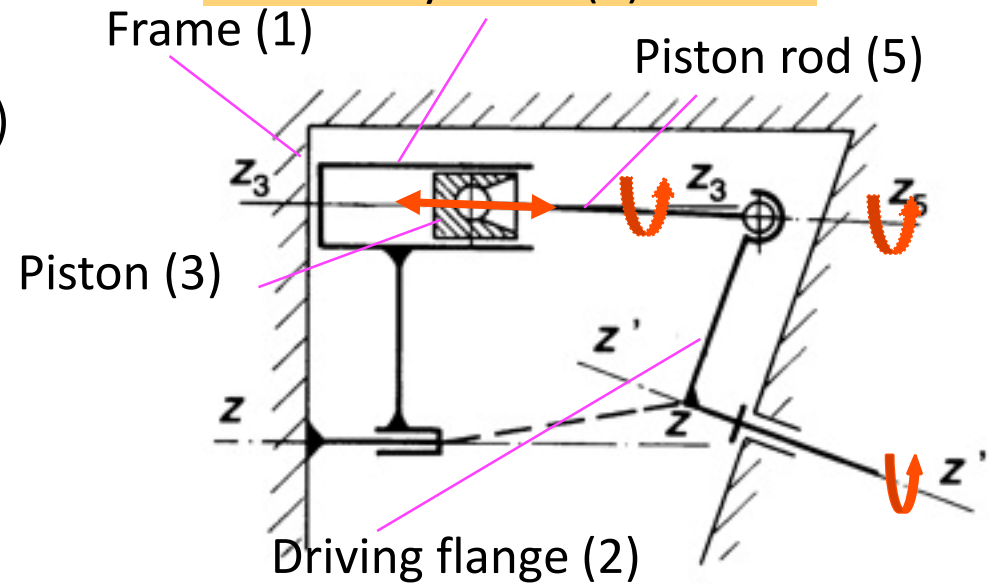


# Αντλίες αξονικών εμβόλων bent axis-κινηματικά

Four link 3D mechanism



Five link 3D mechanism  
Cylinder (4)



Assuming a fixed connection between link 2 and link 4, achieved by synchronization

mechanism has finally two degrees of freedom

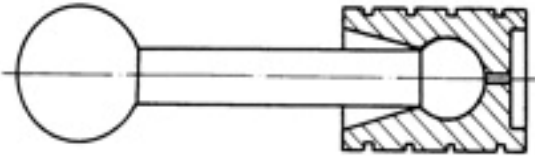
Piston can rotate about  $z_3$ -axis

the mechanism has finally three degrees of freedom

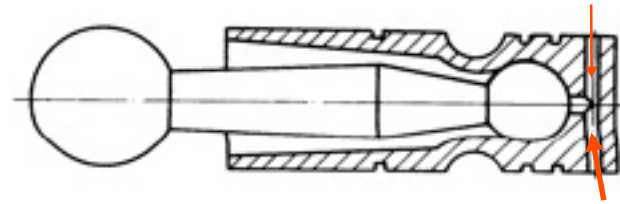
Piston can rotate about  $z_3$ -axis and piston rod can rotate about  $z_5$ -axis

# Σχεδιασμός εμβόλου

Short piston with piston rod



Long piston with piston rod



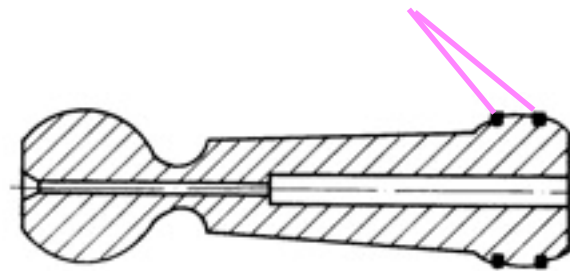
Synchronization by universal joint or bevel gear

Synchronization by pistons or piston rods

Spherical piston with piston ring



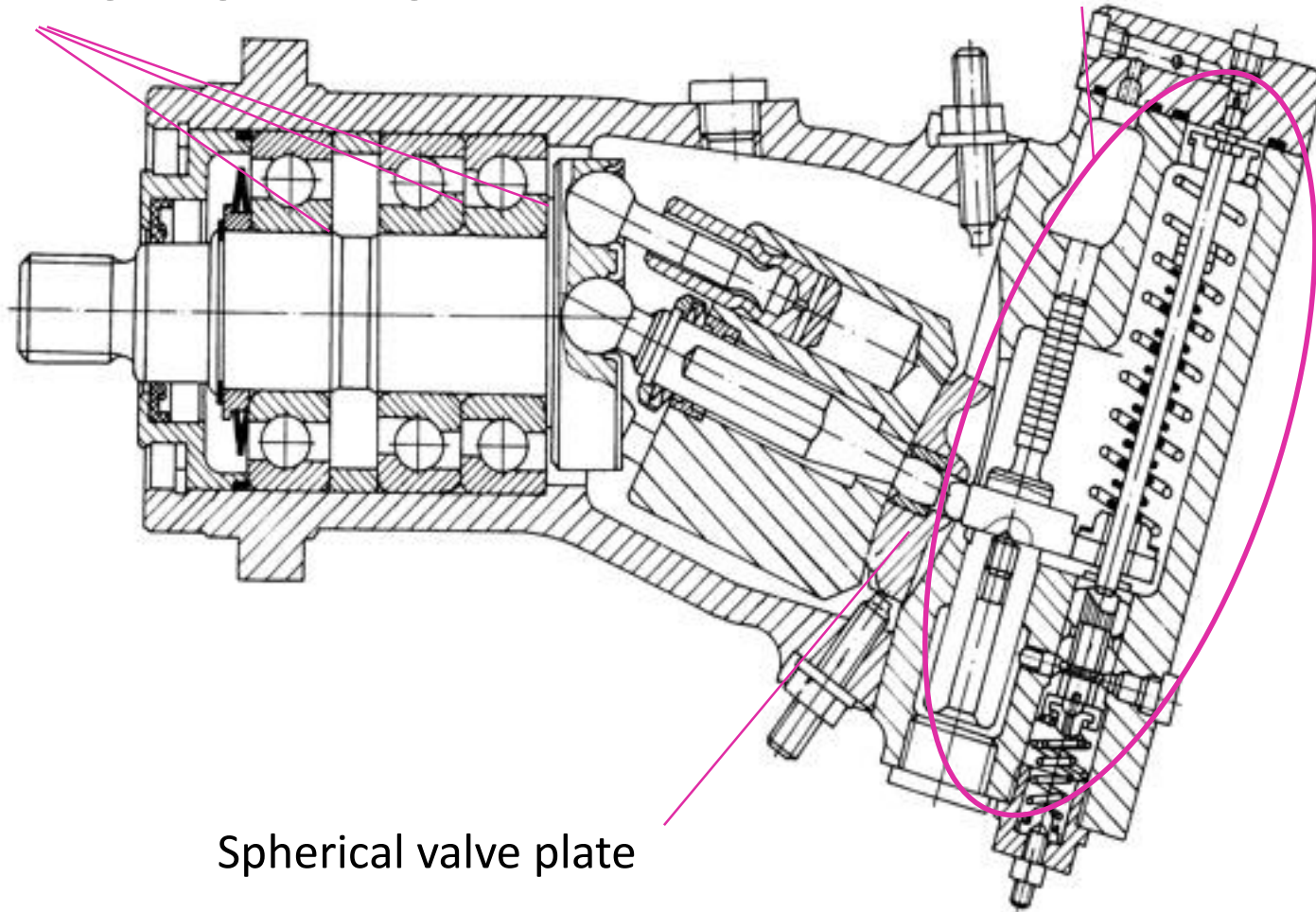
Conical piston with piston rings



# Παράδειγμα σχεδιασμού

Driving flange bearings

Pump control device



Spherical valve plate

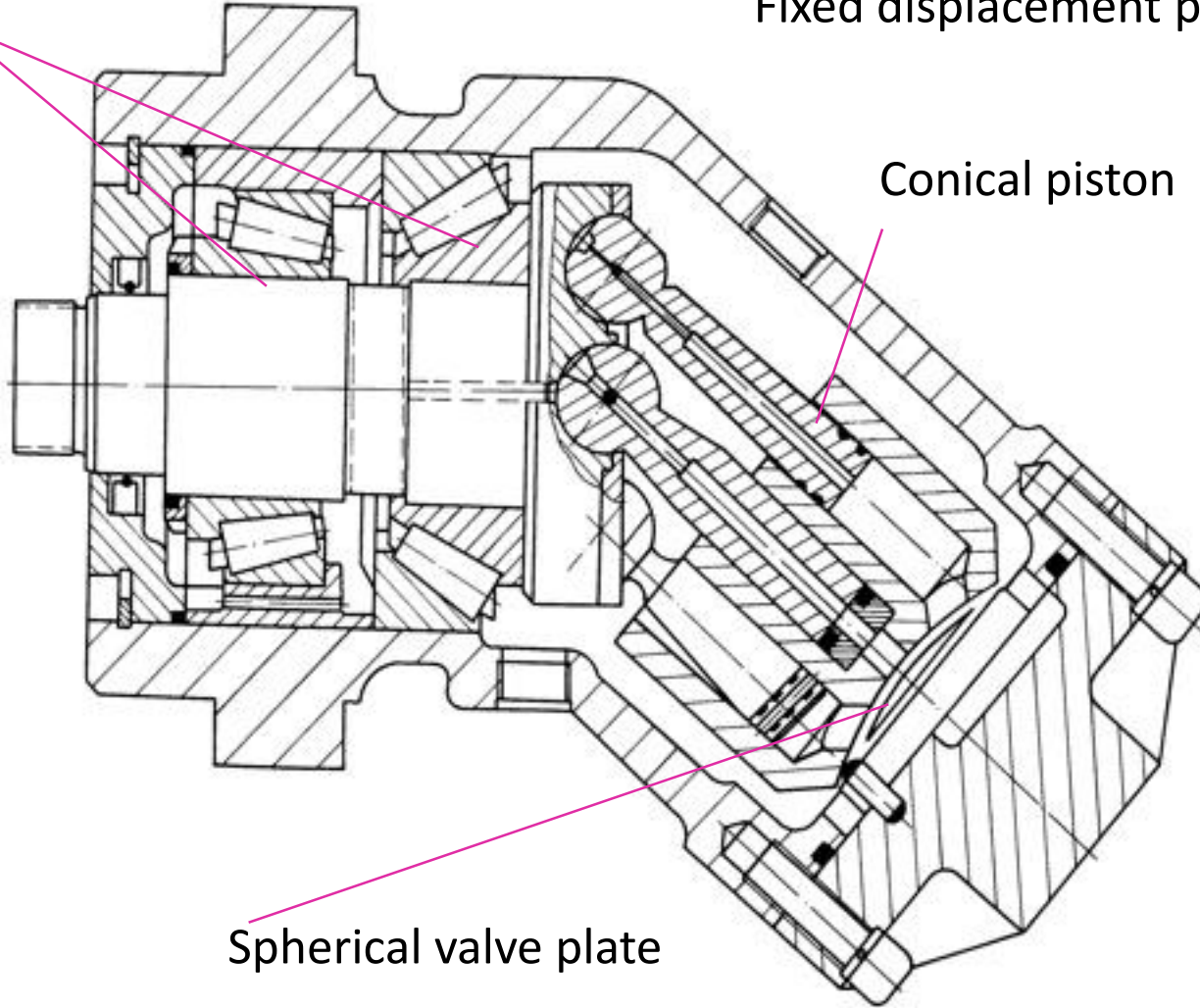
# Παράδειγμα σχεδιασμού

Driving flange bearings

Fixed displacement pump

Conical piston

Spherical valve plate

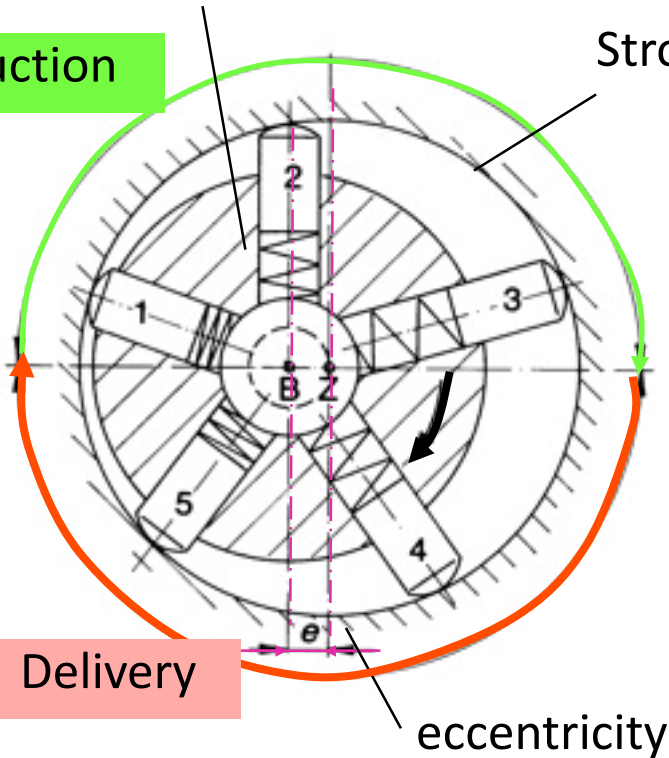


# Αντλίες ακτινικών εμβόλων

with external piston support  
Rotating cylinder body

Suction

Stroke ring



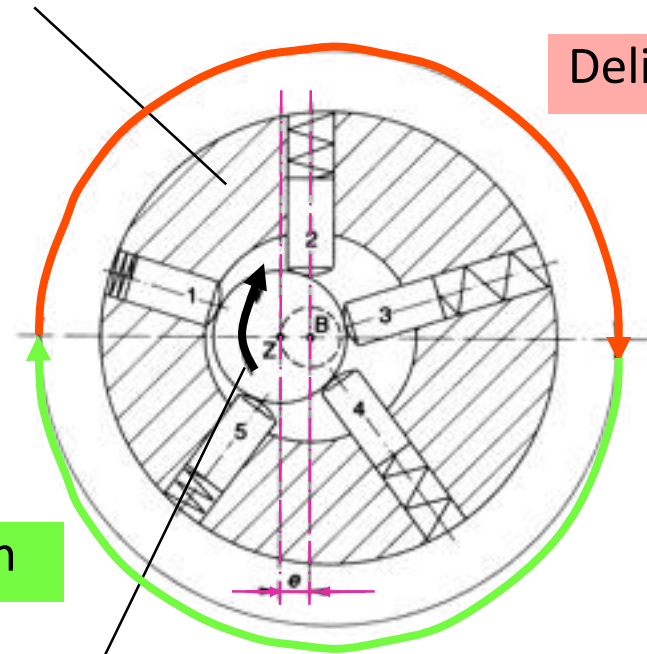
Delivery

eccentricity

with internal piston support  
Stationary cylinder body

Delivery

Suction



Rotating cam or crankshaft



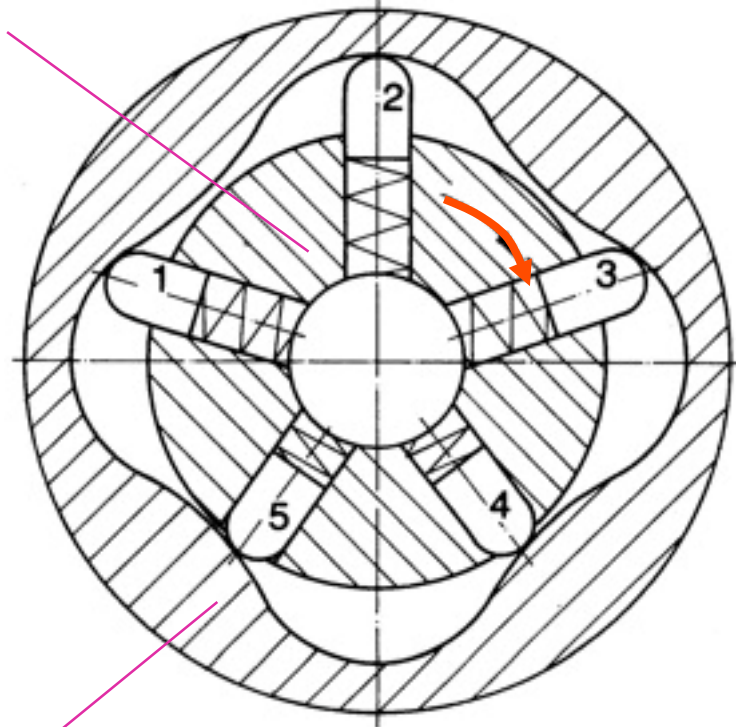
Displacement volume adjustable by changing eccentricity  $e$

# Radial Piston Pumps

Multiple stroke radial piston pumps

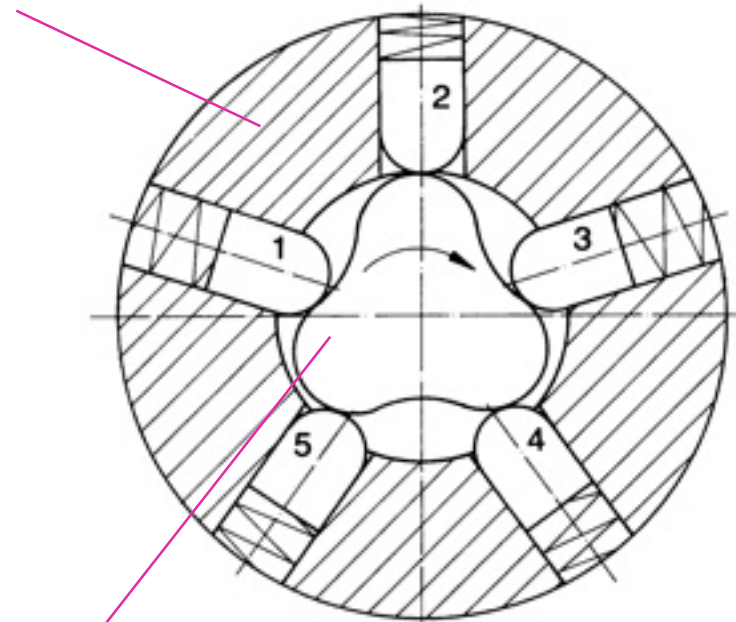
with external piston support

Rotating cylinder body



with internal piston support

Stationary cylinder body



Stationary stroke ring

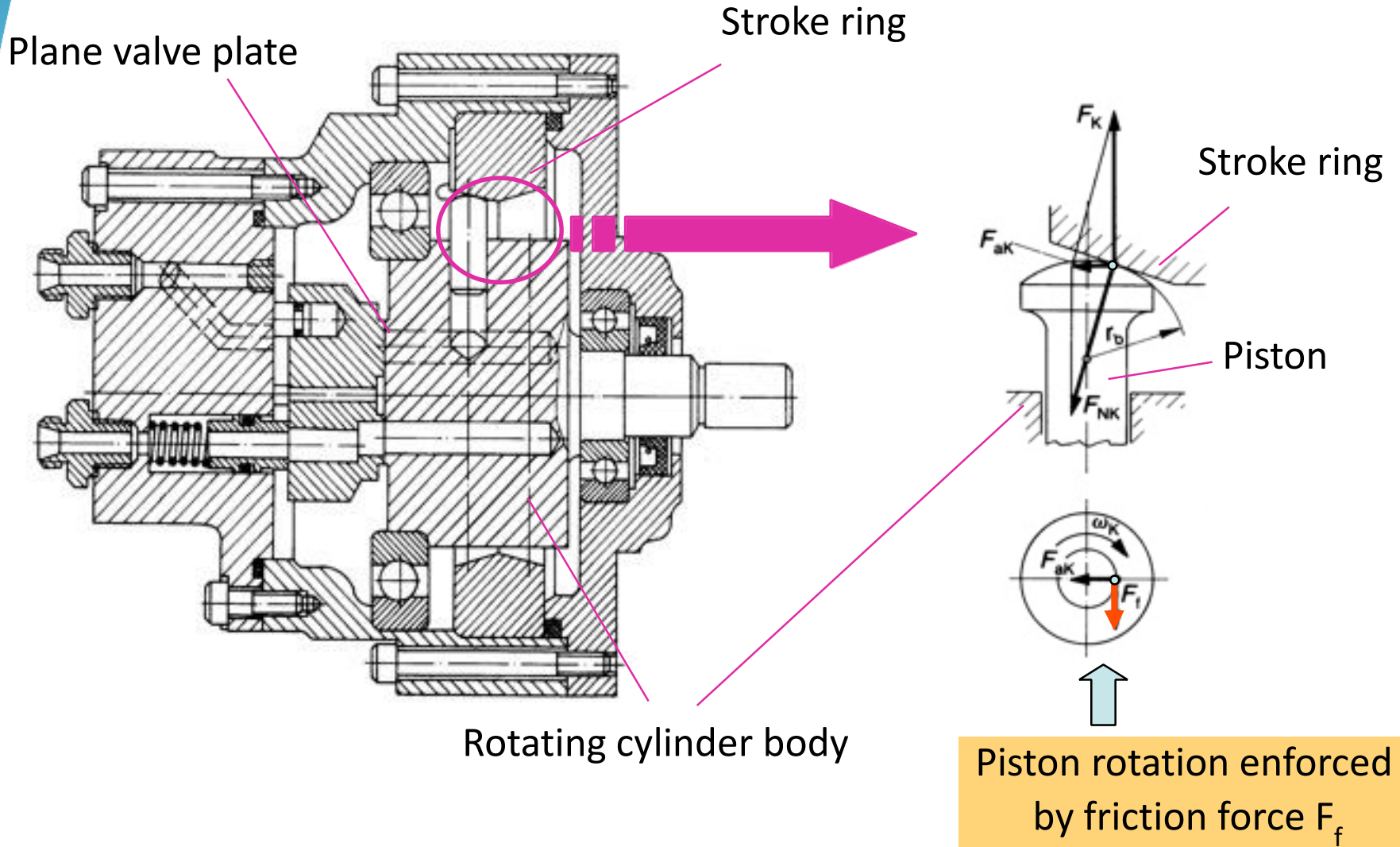
Rotating cam



Only fixed displacement pumps realizable!



# Εξωτερική υποστήριξη εμβόλων



Piston support on outer stroke ring

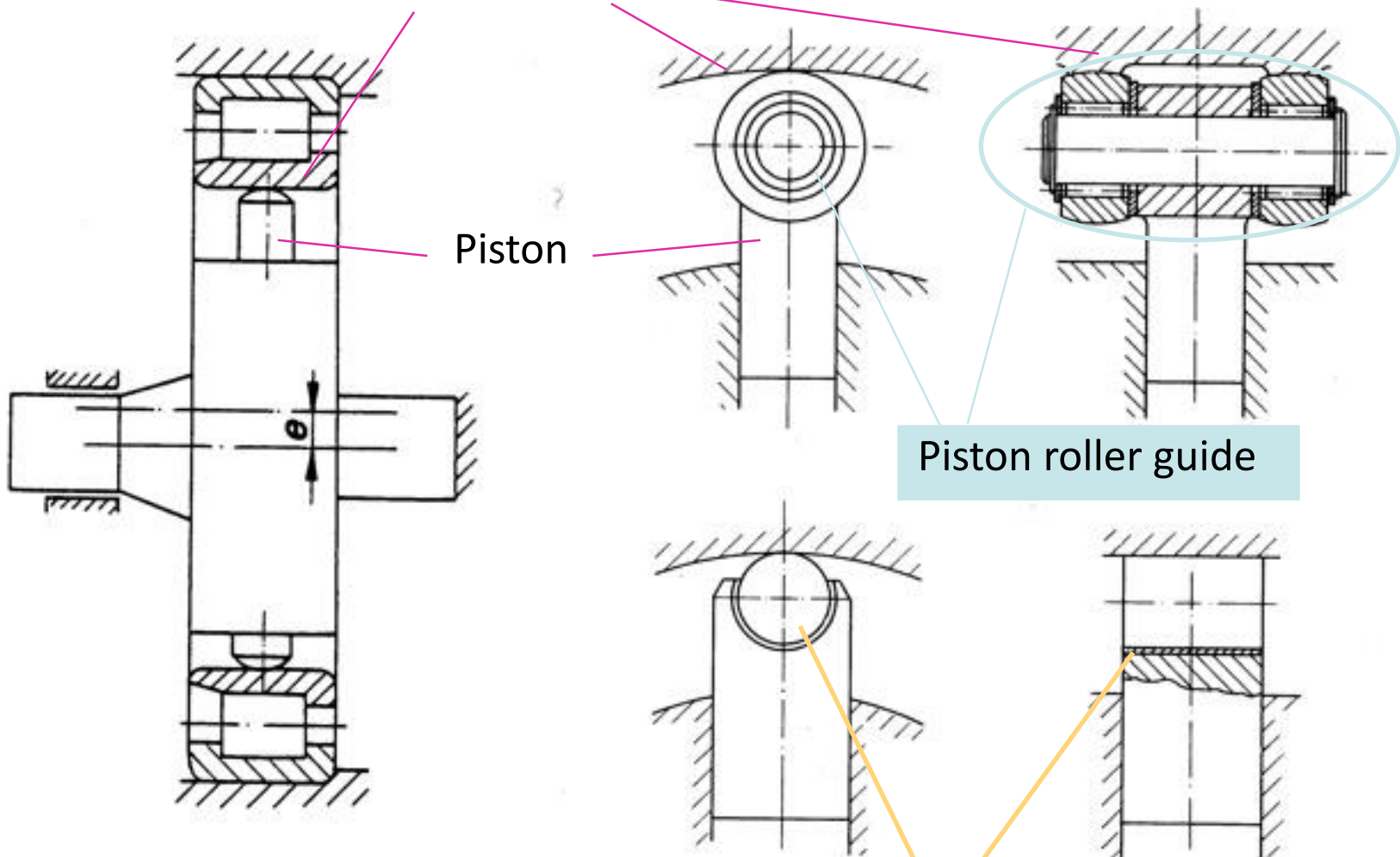
## Εξωτερική υποστήριξη εμβόλων

Stroke ring

Piston

Piston roller guide

Piston sliding bearing



Stroke ring borne in roller bearings

# Εξωτερική υποστήριξη εμβόλων

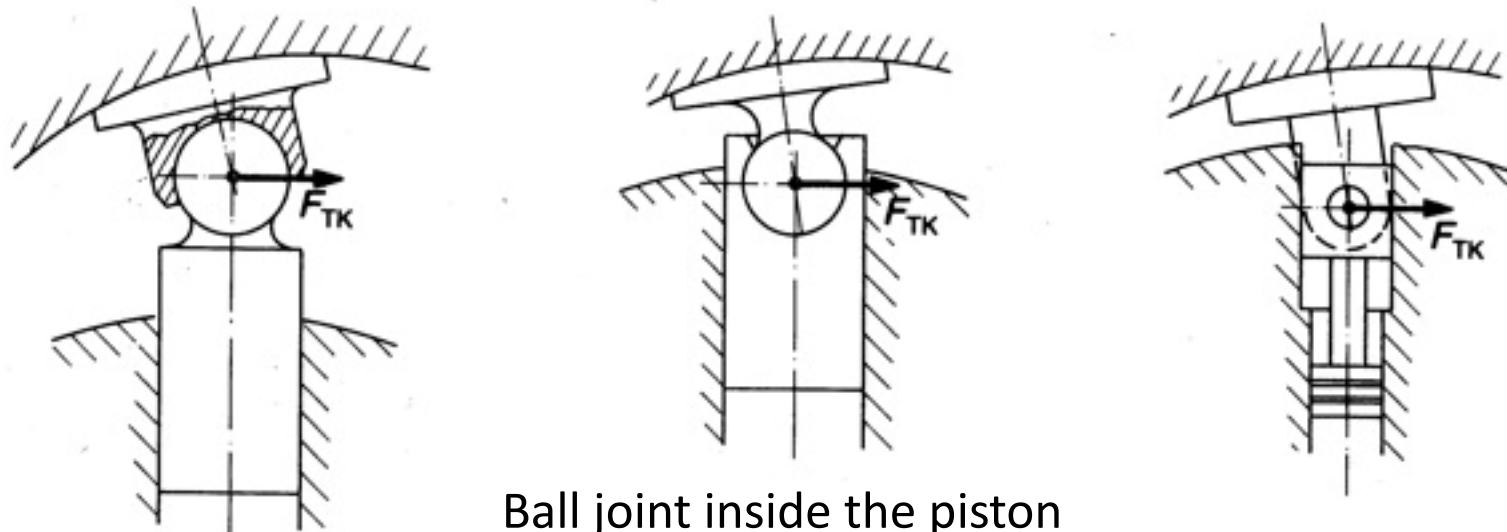
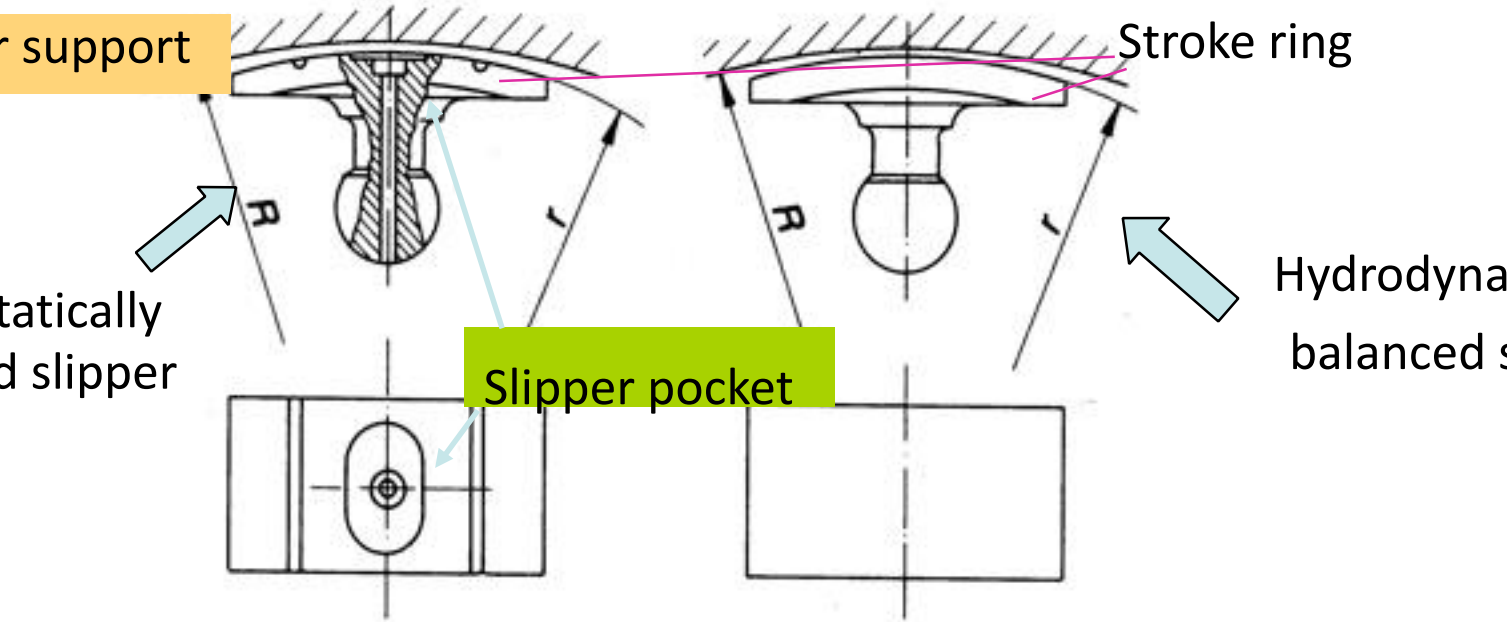
Slipper support

Stroke ring

Hydrostatically balanced slipper

Hydrodynamically balanced slipper

Slipper pocket

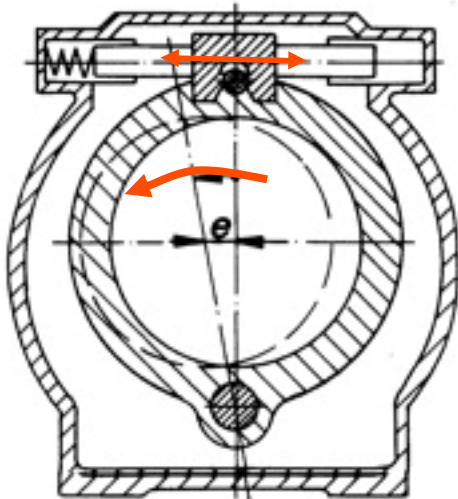
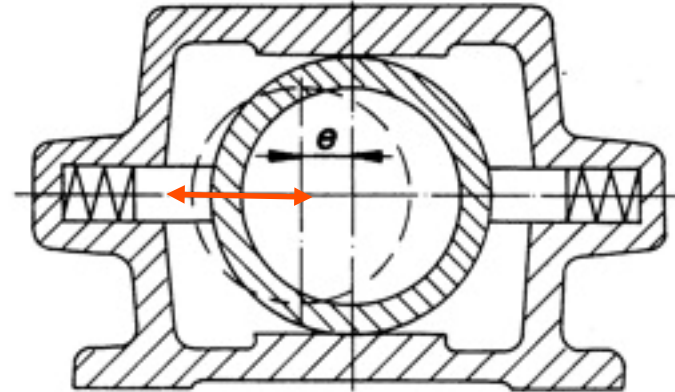
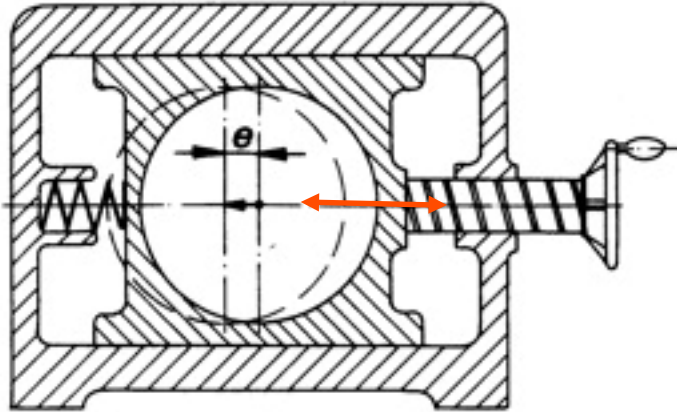


Ball joint inside the piston

# Εναλλακτική υποστήριξη

Using a sliding carriage

supported using line contact



Stroke ring mounted on a pivot

Change of eccentricity by pivoting the stroke ring about pivot axis

# Παράδειγμα σχεδιασμού

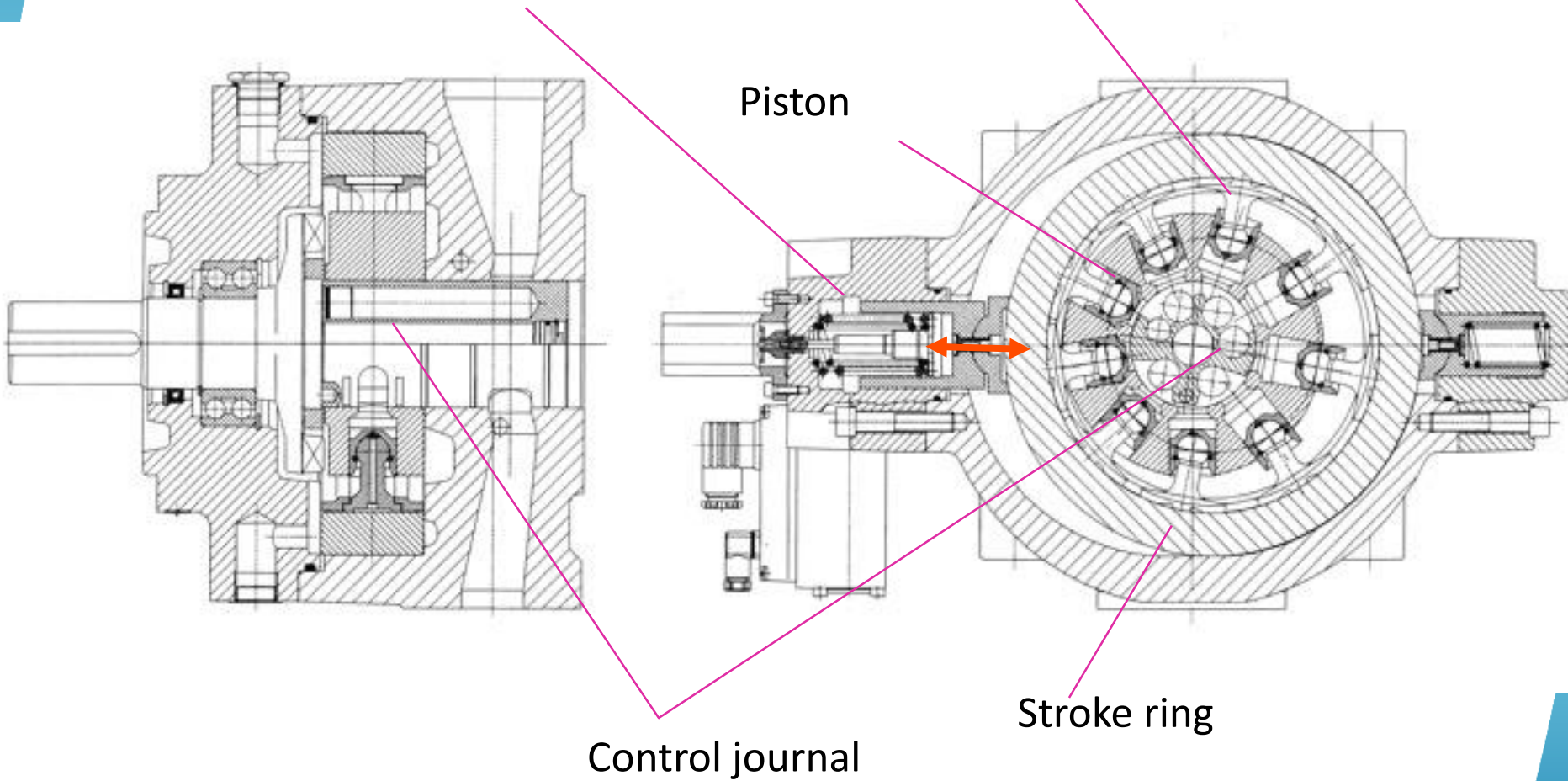
Pump control system

Slipper

Piston

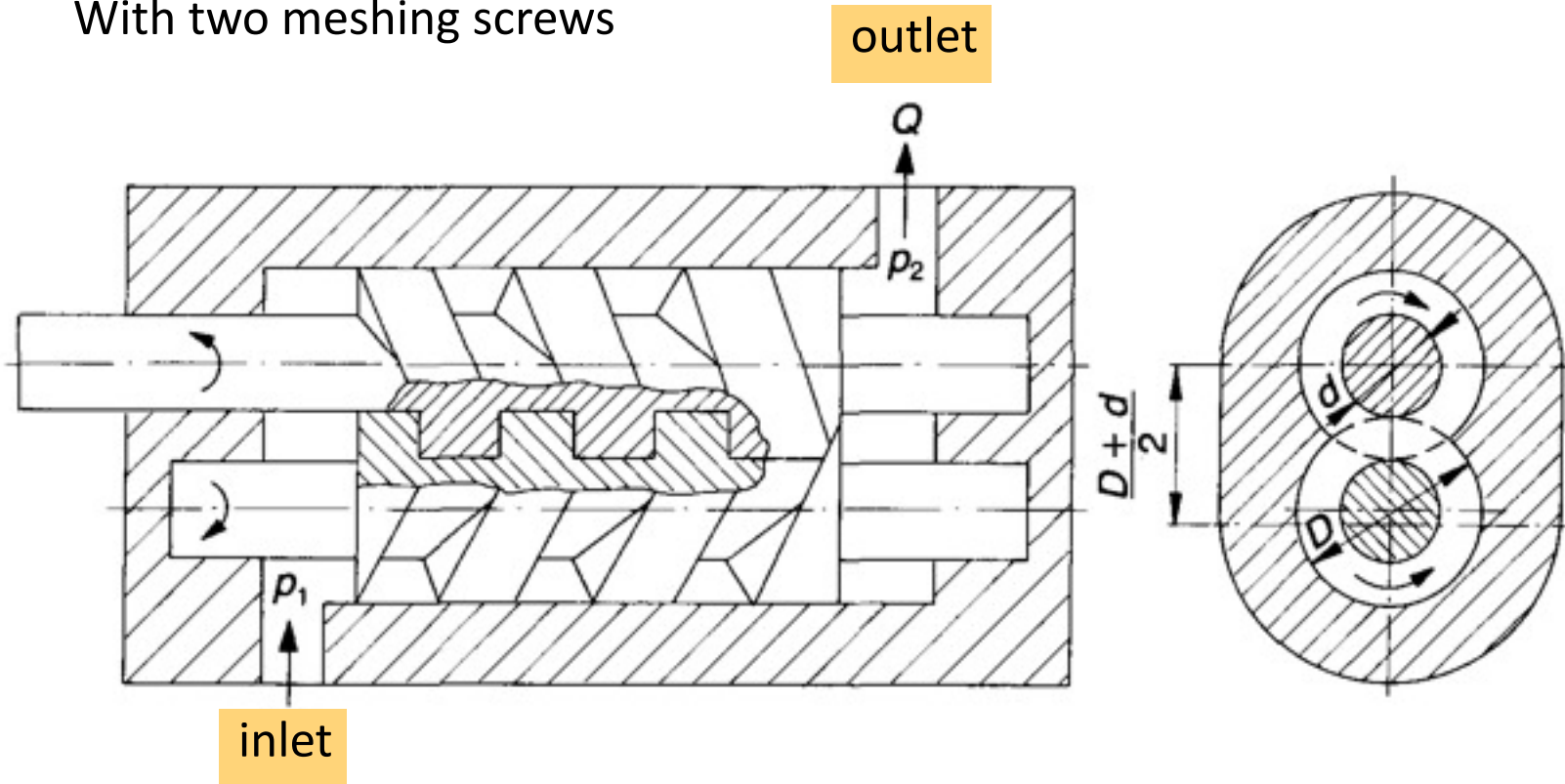
Control journal

Stroke ring

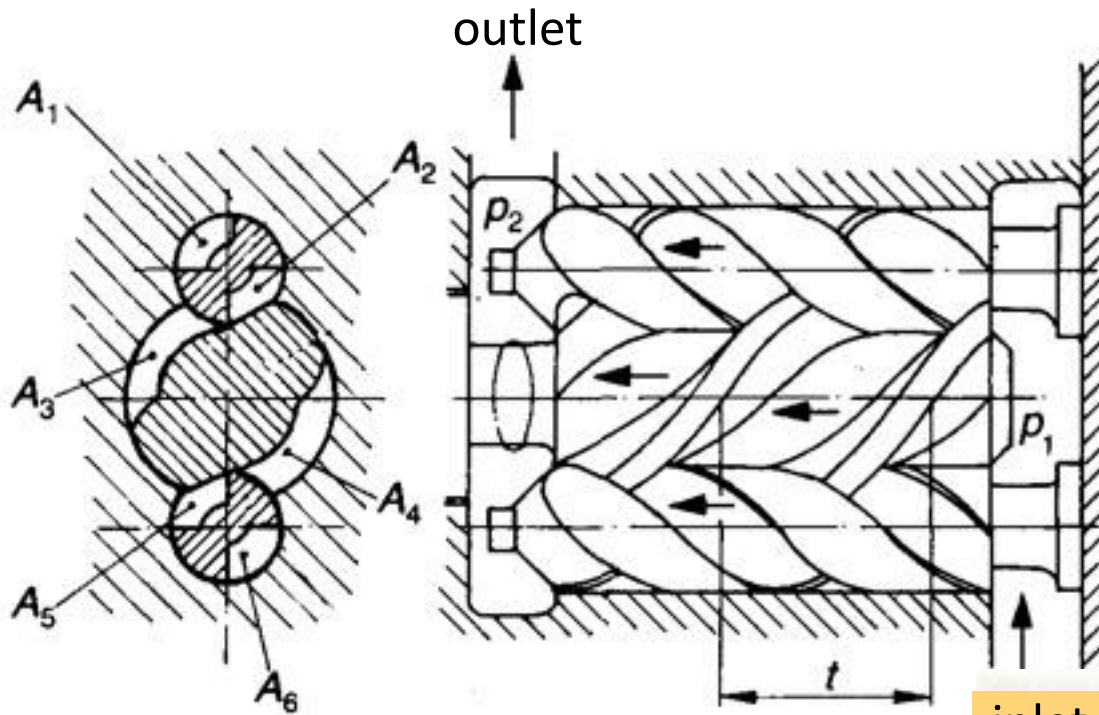


# Κοχλιωτή αντλία

With two meshing screws

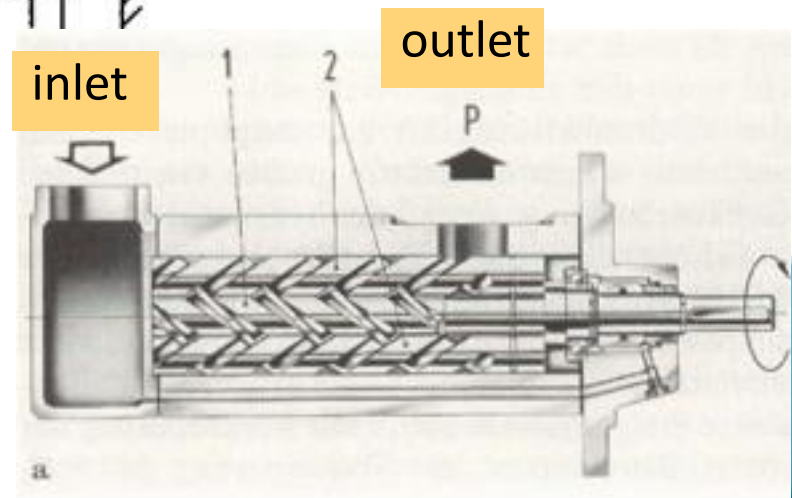


# Κοχλιωτή αντλία



t...thread pitch

With three meshing screws



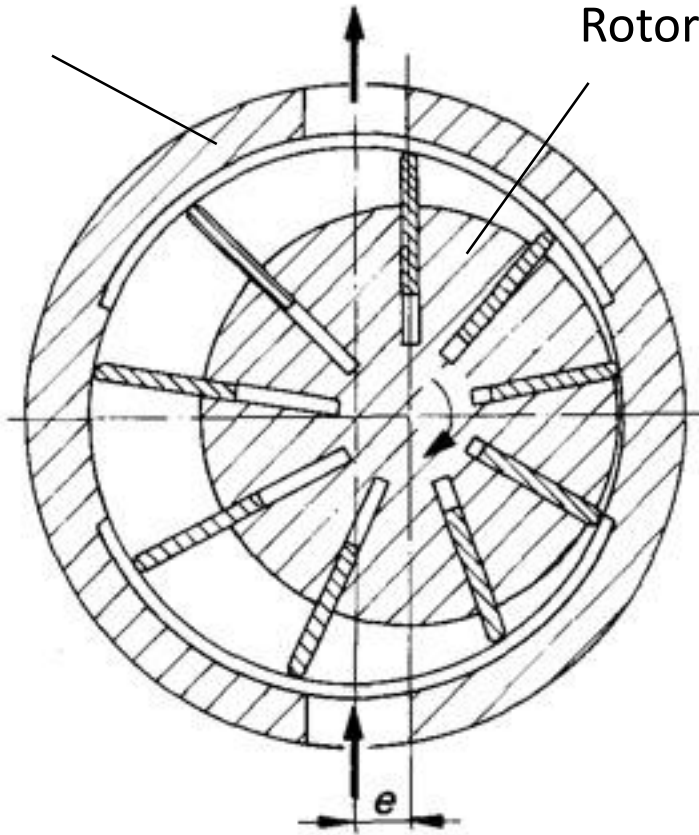
# Κοχλιωτή αντλία

- Σελίδα 14 διάλεξη 7



# Πτερυγιοφόρος αντλία

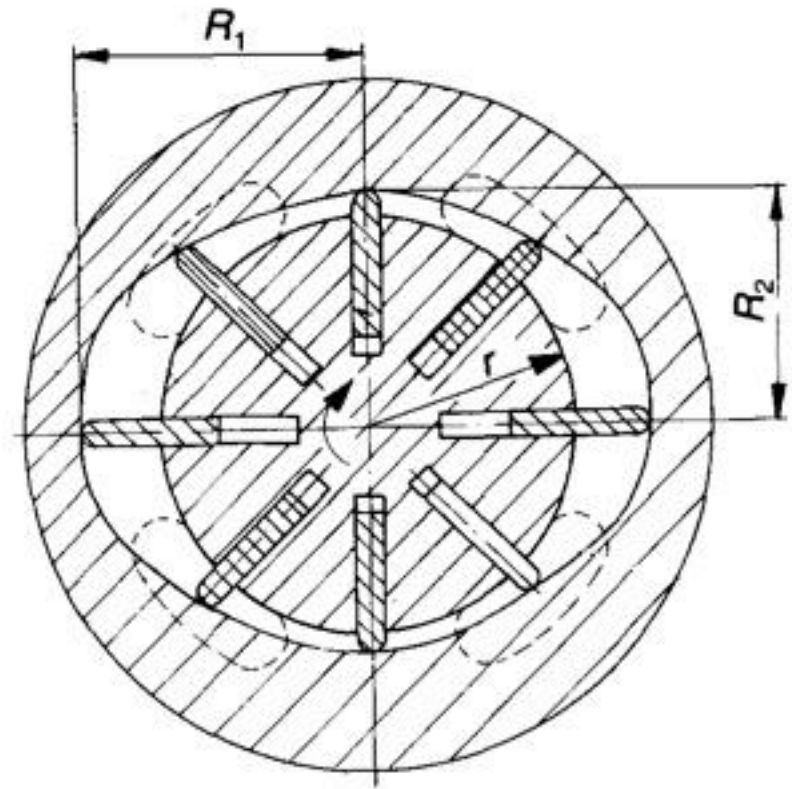
Unbalanced vane pump  
Stator



Rotor

Fixed and variable pump design

Balanced vane pump



Only fixed displacement pump

# Πτερυγιοφόρος αντλία

Single stroke vane pump – variable displacement volume

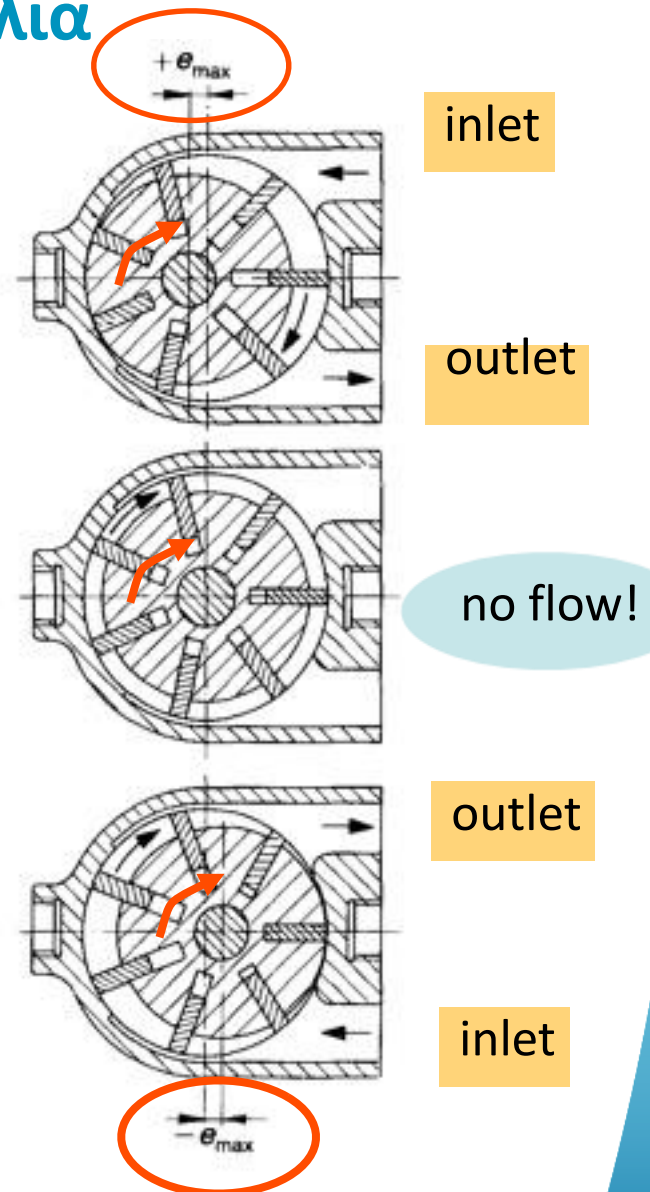
Overcenter pump – the direction of flow can be reversed by change of eccentricity, i.e. without changing the direction of rotation of the drive shaft

Relatively high friction between axial moveable vanes and rotor & between vanes and stator

Large radial forces exerted on the rotor

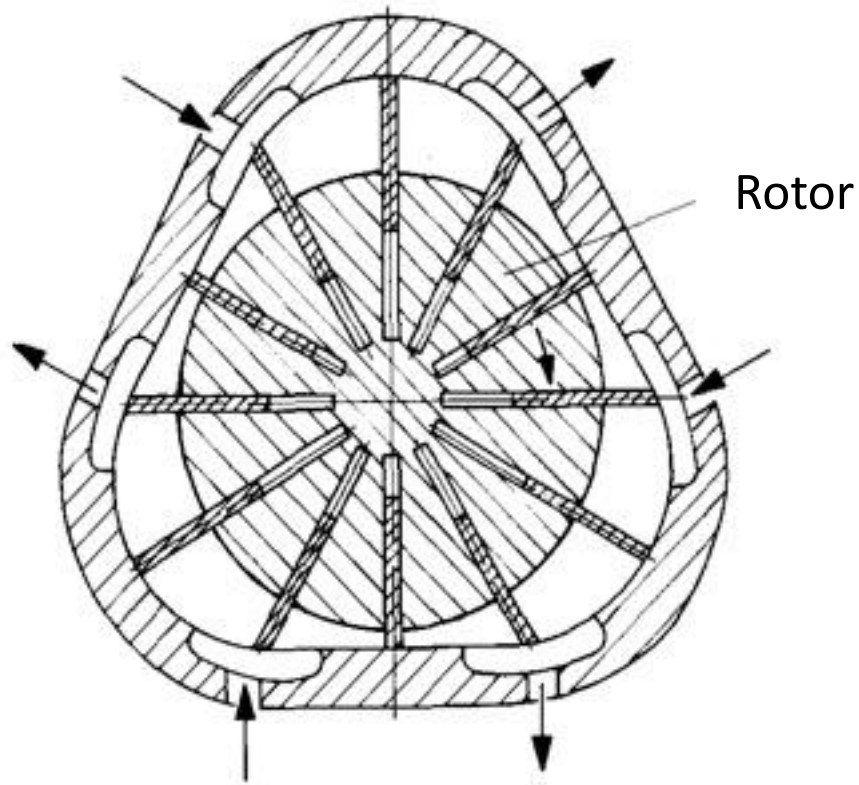


Limitation of max. operating pressure (20 MPa)

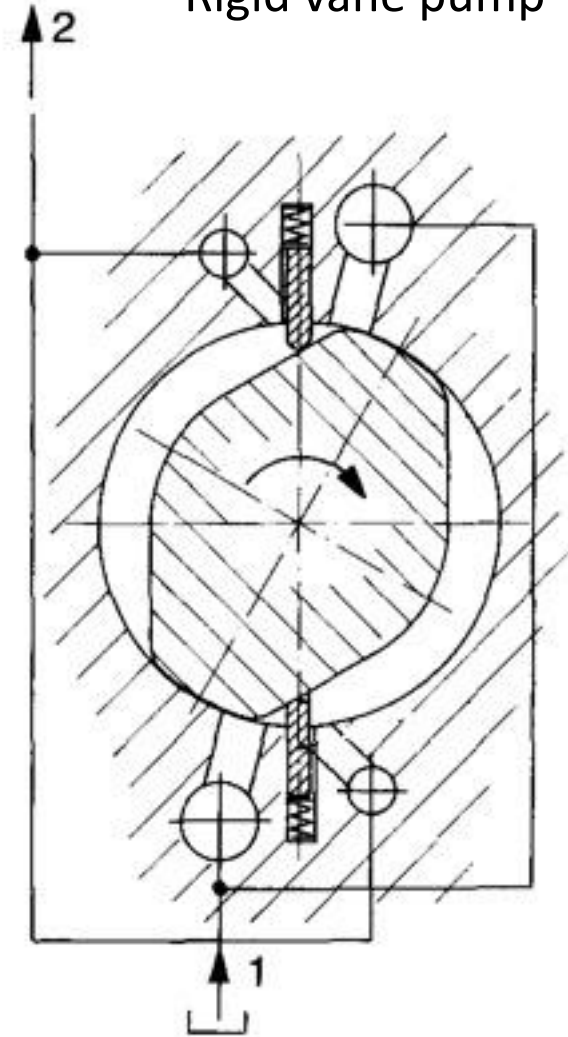


# Πτερυγιοφόρος αντλία

Multiple stroke vane pump



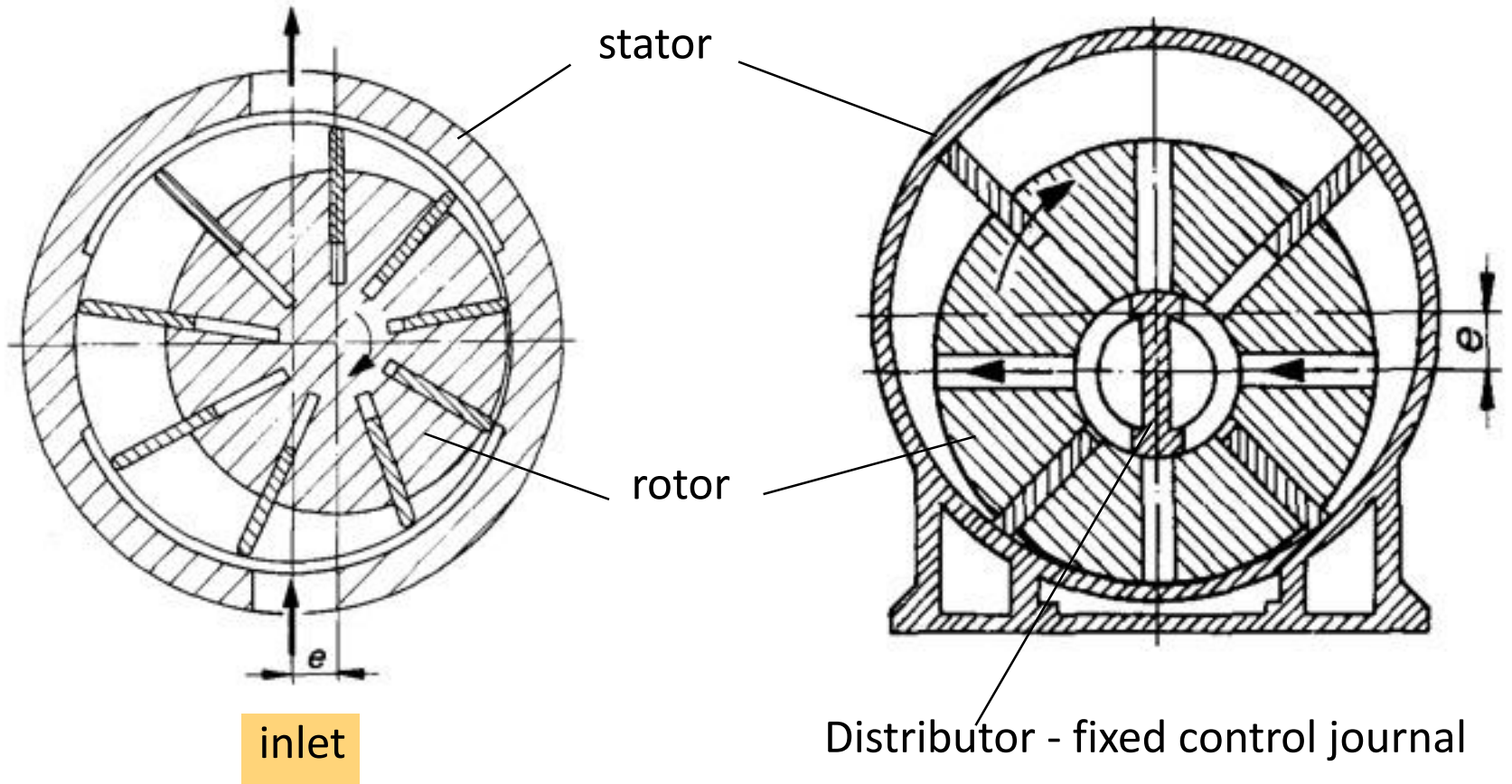
Rigid vane pump



# Πτερυγιοφόρος αντλία

External fluid distribution  
outlet

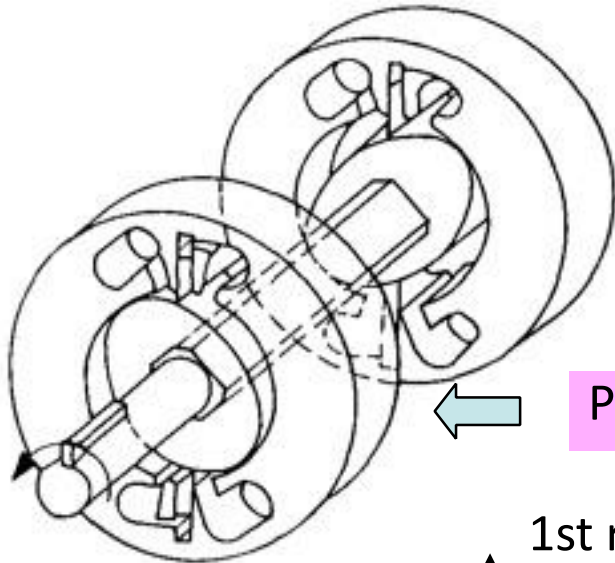
Internal fluid distribution



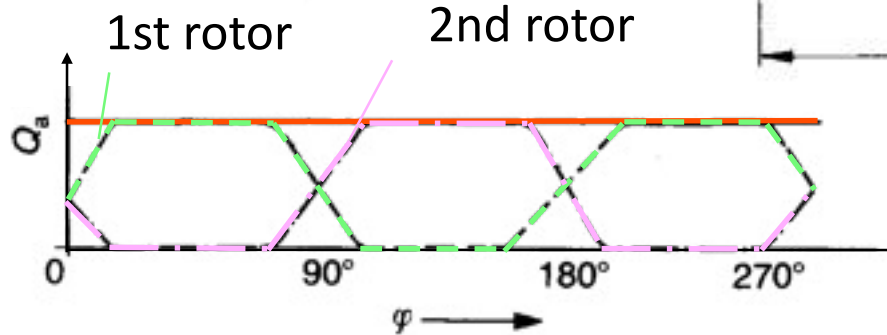
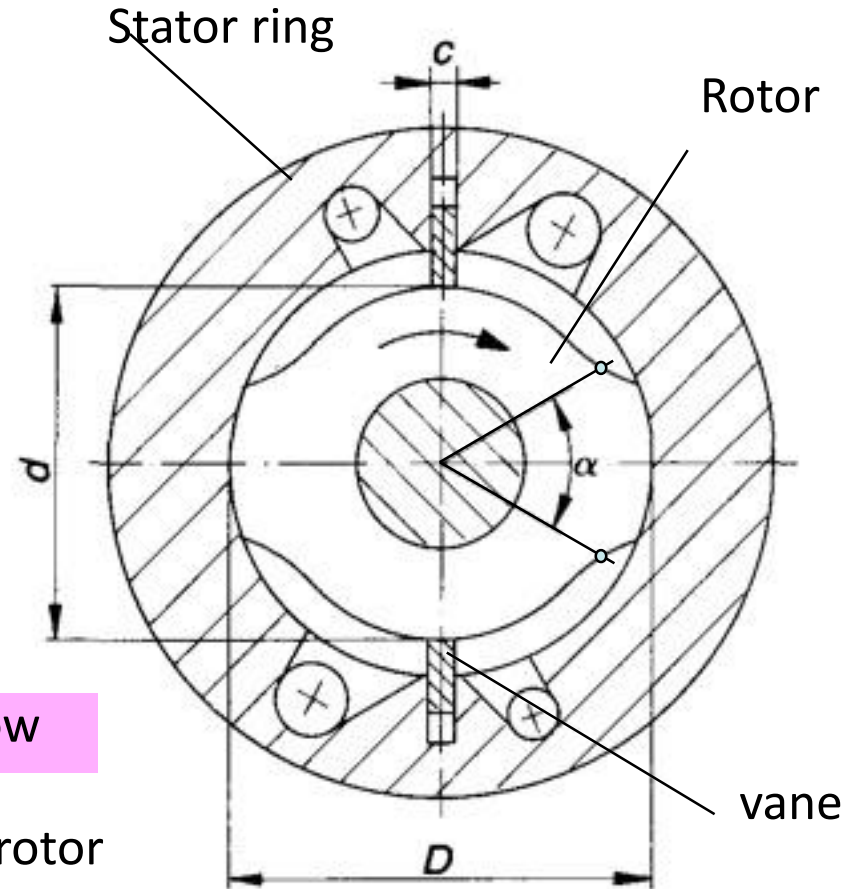
# Πτερυγιοφόρος αντλία

Displacement volume:

$$V_s = 2 \cdot \frac{\pi \cdot (D^2 - d^2)}{4} \cdot \frac{180 - \alpha}{180} \cdot b - 2 \cdot \left( \frac{D}{2} - \frac{d}{2} \right) \cdot c$$



Pulsation free flow



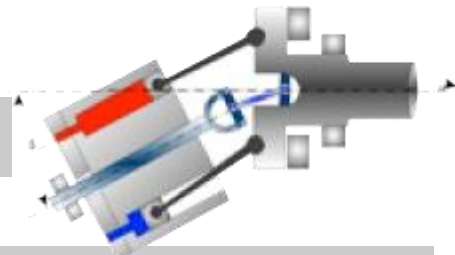
# Μηχανές θετικής μετατόπισης

Piston Machines

Axial Piston Machines



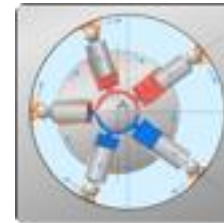
Swash Plate Machines



Bent Axis machines

In-line Piston Machines

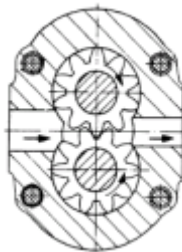
Radial Piston Machines



with external piston support

with internal piston support

Gear Machines

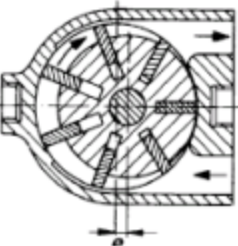
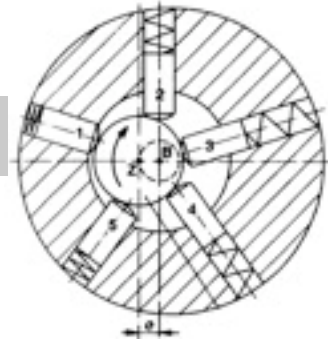


External Gear

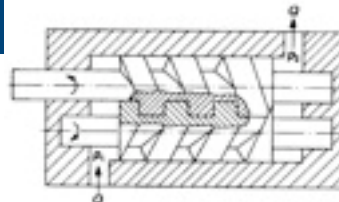
Internal Gear



Annual Gear



Vane Machines



Screw Machines

others

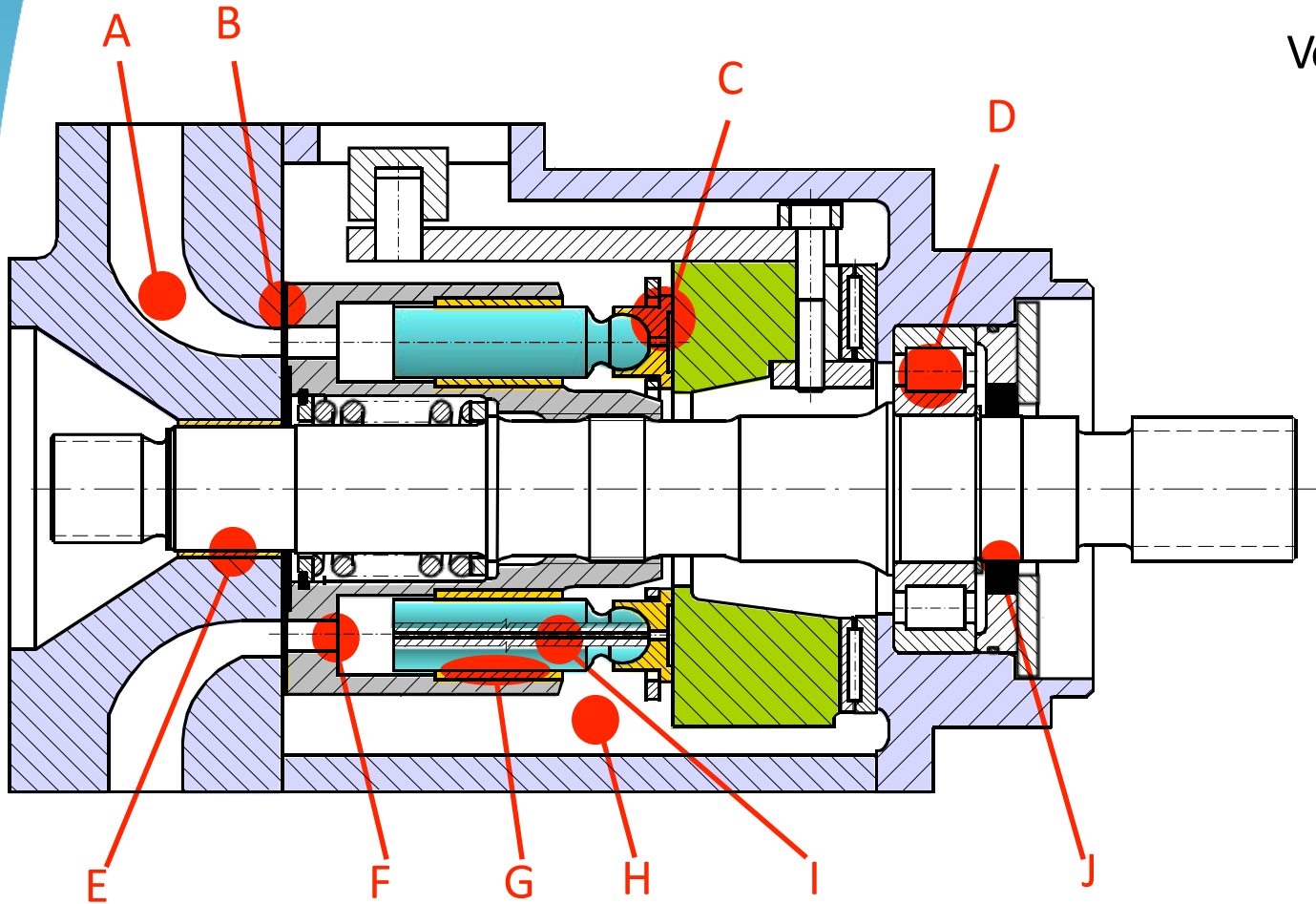
Fixed displacement machines



Variable displacement machines

# Απώλειες στις αντλίες θετικής μετατόπισης

Axial piston machine – swash plate design



Volumetric Losses

B C F G I

Torque Losses

A B C D E

G H J

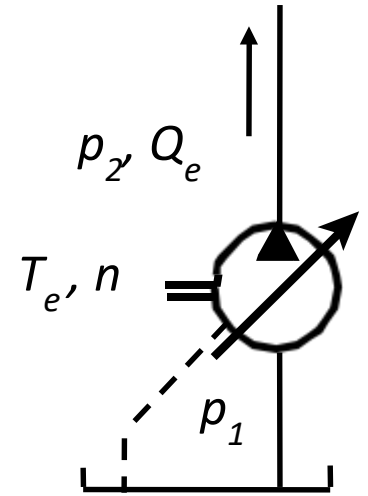
## Βαθμοί απόδοσης

Volumetric efficiency:  $\eta_v = \frac{Q_e}{n \cdot V_i}$

where  $V_i$ .... represents the derived displacement volume

Torque efficiency  
(hydraulic-mechanical efficiency):

$$\eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e}$$



$$\Delta p = p_2 - p_1$$

Total efficiency:

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm}$$



The derived displacement volume can only be determined by measurement



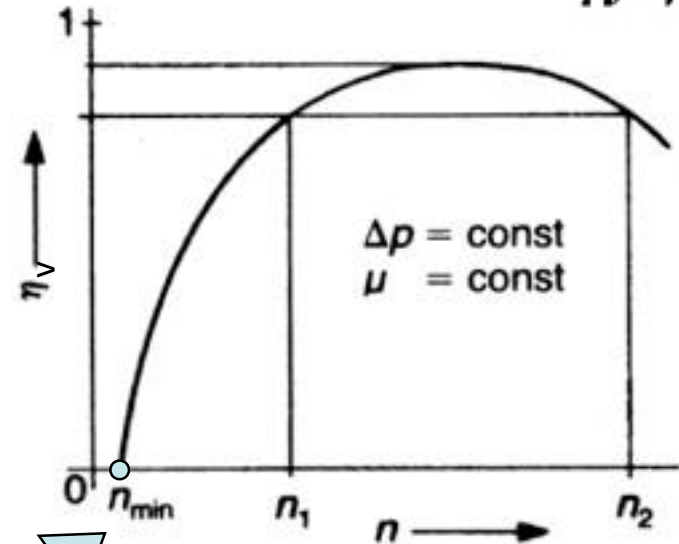
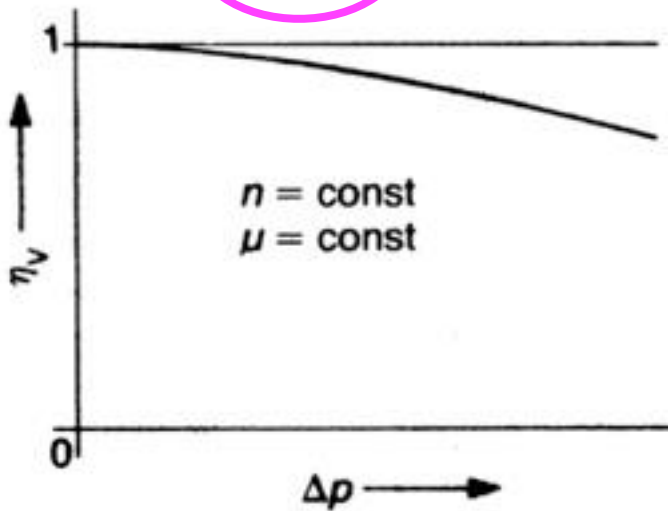
# Ογκομετρικός βαθμός απόδοσης

Volumetric efficiency

$$\eta_v = f(\Delta p, n, V_i, \mu)$$

$$\eta_v = \frac{Q_e}{n \cdot V_i}$$

$$Q_e = \alpha \cdot V_{max} \cdot n - Q_s$$



$$\alpha = \frac{V}{V_{max}}$$

$$V_i \cdot n_{min} = Q_s$$

Dynamic viscosity of fluid:

$$\mu = f(\theta, p) \quad [\text{Pa} \cdot \text{s}]$$

# Ουκομετρικός βαθμός απόδοσης

Volumetric efficiency

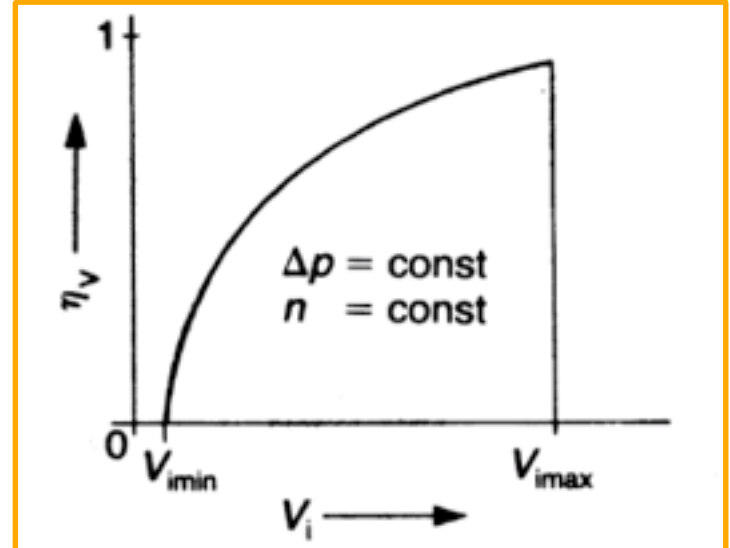
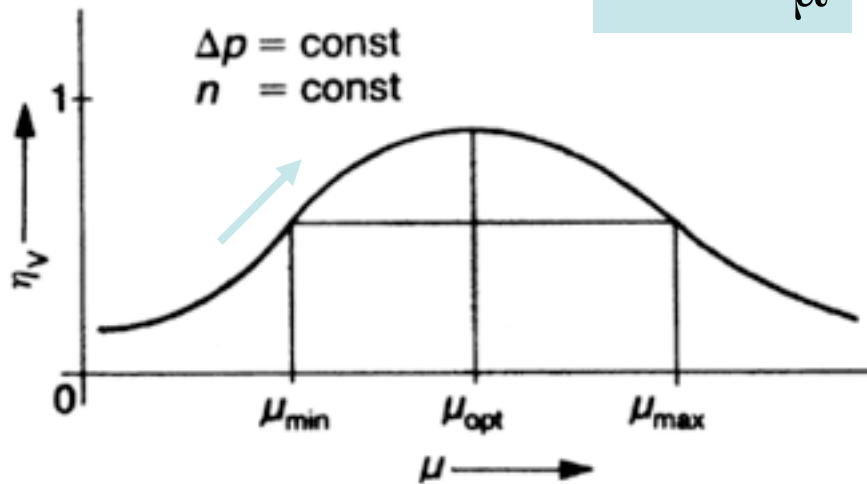
$$\eta_v = f(\Delta p, n, V_i, \mu)$$

$$\eta_v = \frac{Q_e}{n \cdot V_i}$$

$$Q_e = \alpha \cdot V_{max} \cdot n - Q_s$$

$$Q_{SL\mu} \approx \frac{1}{\mu}$$

Variable displacement pump



Typical values of dynamic viscosity used in displacement machines:

$$0.0435 \text{ Pa} \cdot \text{s} \div 0.0087 \text{ Pa} \cdot \text{s}$$

with: Kinematic viscosity  $\nu$  [cSt, mm<sup>2</sup>/s]

and

$$\mu = \nu \cdot \rho$$

$$\nu = 10 \text{ mm}^2 \cdot \text{s}^{-1} \div 50 \text{ mm}^2 \cdot \text{s}^{-1} \text{ with } \rho = 870 \text{ kg} \cdot \text{m}^{-3}$$

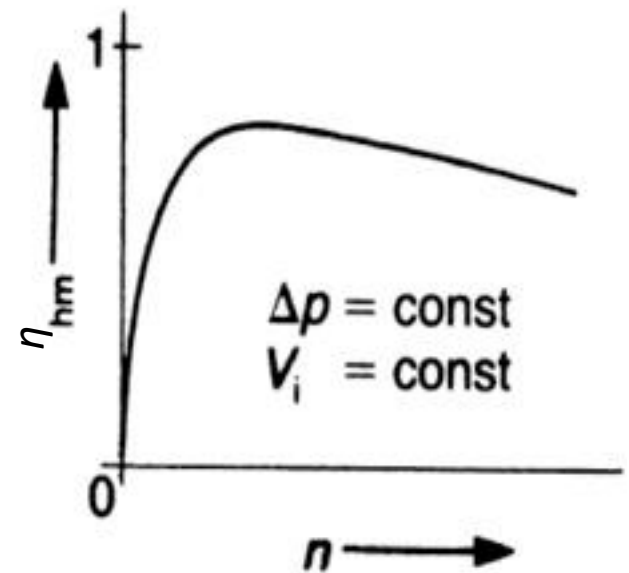
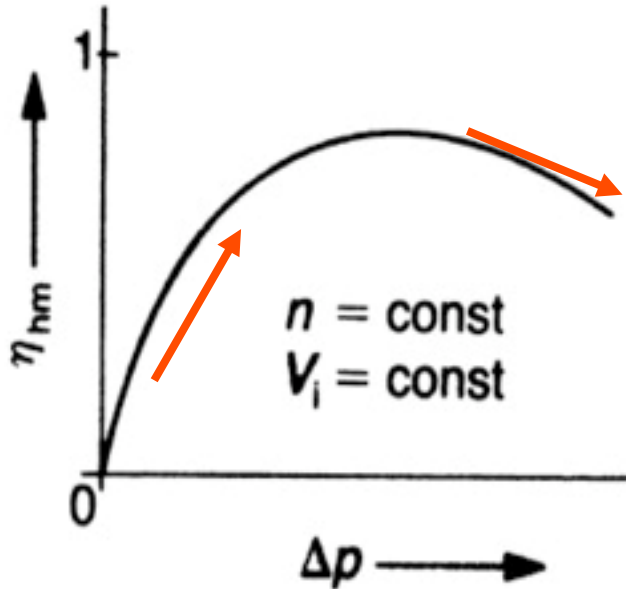
## Απόδοση ροπής

$$\eta_{hm} = f(\Delta p, n, V_i, \mu)$$

$$\eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e} = 1 - \frac{T_s}{T_e}$$

$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{\max}}{2 \cdot \pi} + T_s$$

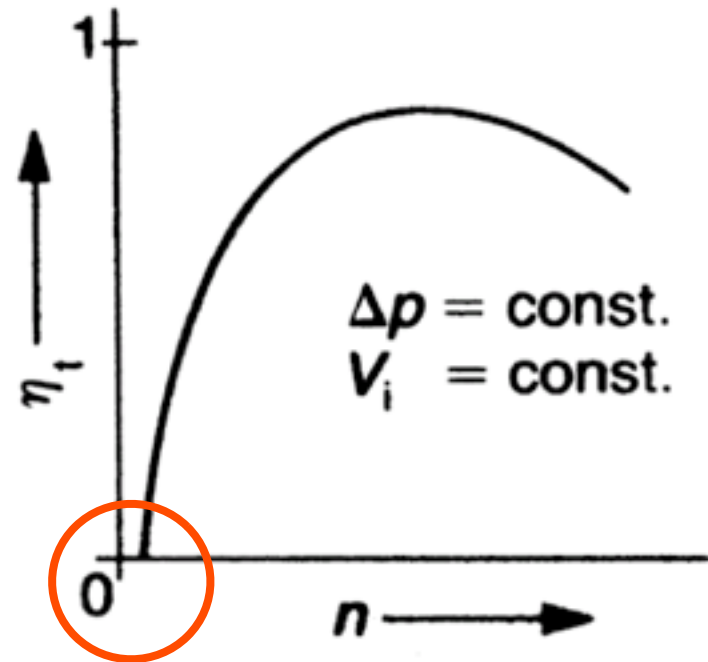
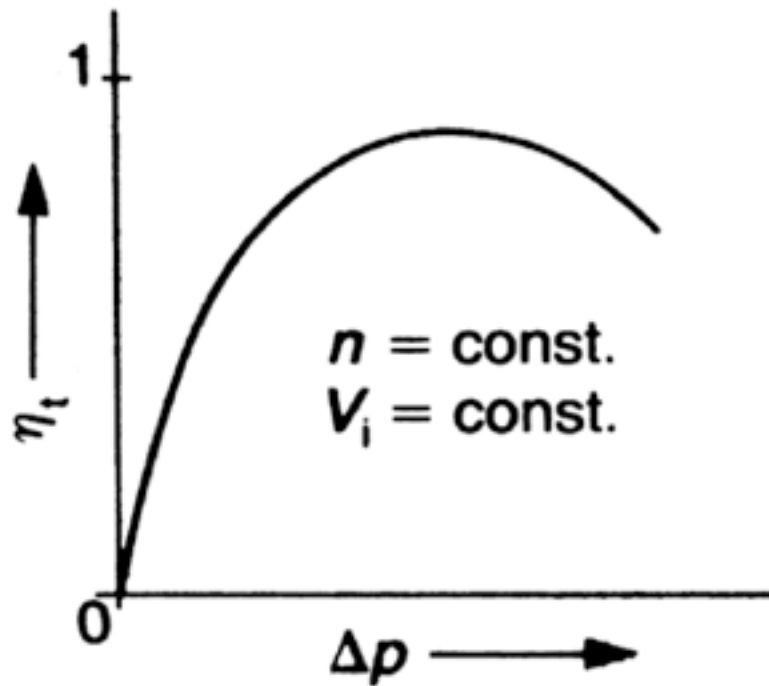
$$T_s = T_{S\mu} + T_{S\rho} + T_{Sp} + T_{Sc} = C_\mu \cdot \mu \cdot n + C_\rho \cdot \rho \cdot n^2 + C_p \cdot \Delta p + T_{Sc}$$



## Ολικός βαθμός απόδοσης

$$\eta_t = f(\Delta p, n, V_i, \mu)$$

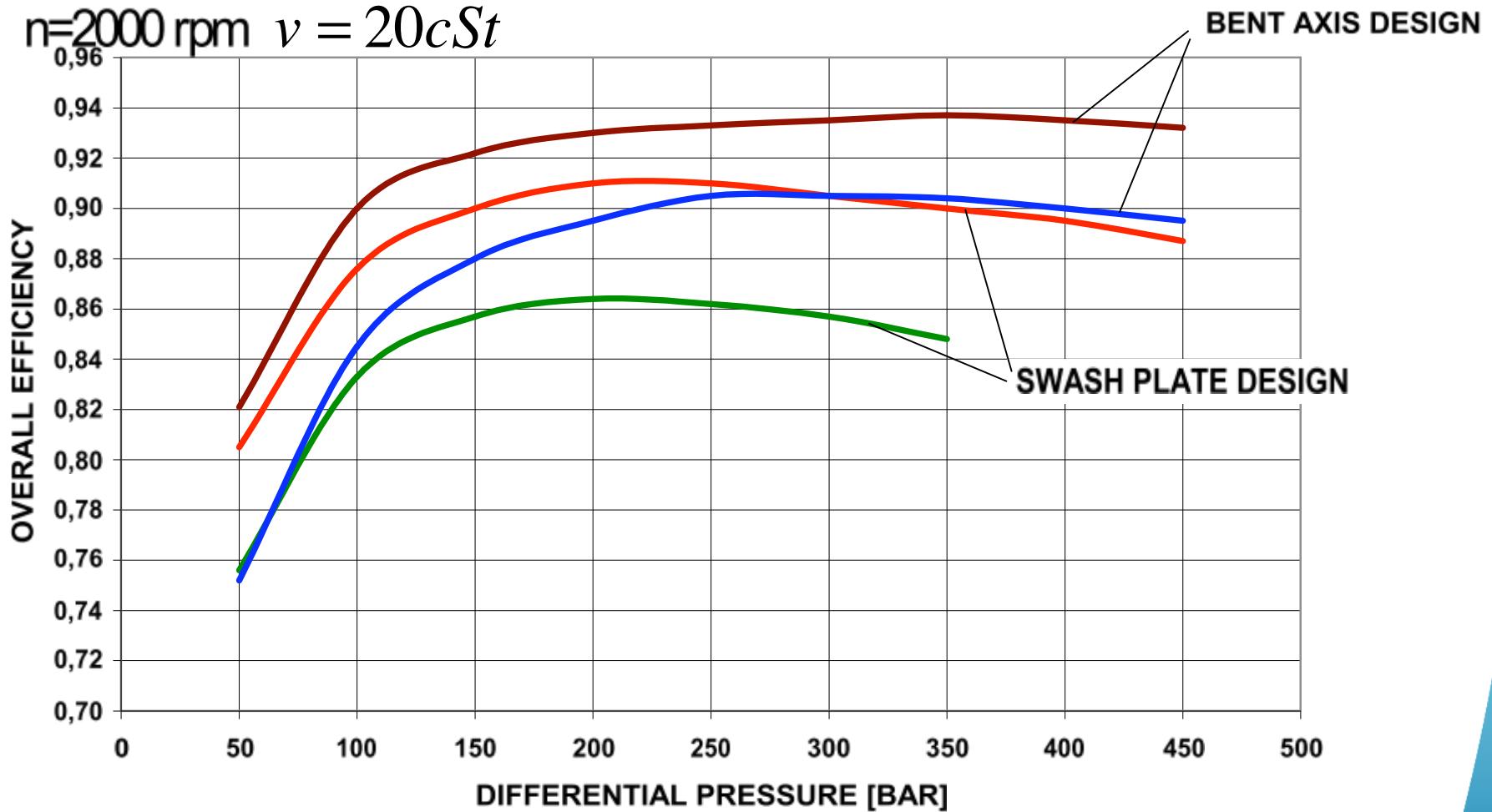
$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm}$$



# Χαρακτηριστικά steady state

## Comparison of Efficiencies of Axial Piston Pumps

$n=2000$  rpm  $\nu = 20cSt$



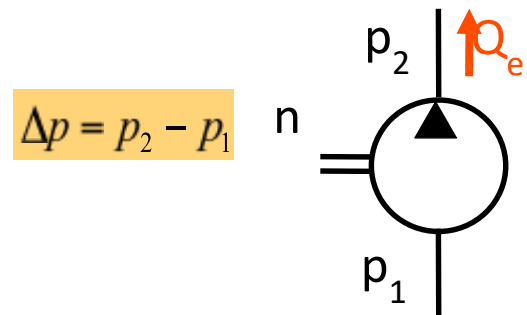
## Παράδειγμα

The displacement pump with a displacement volume of  $V=100 \text{ cm}^3/\text{rev}$  is driven by an electric motor at 1800 rpm. At steady state conditions the pressure difference across the pump is  $\Delta p= 380 \text{ bar}$ . Determine the effective flow rate at the pump outlet  $Q_e$  and the power required to drive this pump. Assume the following values for efficiency:

- volumetric efficiency  $\eta_v=0.87$
- torque efficiency  $\eta_{hm}=0.95$

The effective volume flow rate:

$$Q_e = V \cdot n \cdot \eta_v = 100 \cdot 10^{-6} \text{ m}^3 \cdot 1800 \cdot \frac{1}{60} \cdot \text{s}^{-1} \cdot 0.87 = 0.00261 \text{ m}^3 \cdot \text{s}^{-1} = 156.6 \text{ l} \cdot \text{min}^{-1}$$



The effective torque yields:

$$T_e = \frac{T_i}{\eta_{hm}} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot \eta_{hm}} = \frac{380 \cdot 10^5 \text{ Pa} \cdot 100 \cdot 10^{-6} \text{ m}^3}{2 \cdot \pi \cdot 0.95} = 636.62 \text{ Nm}$$

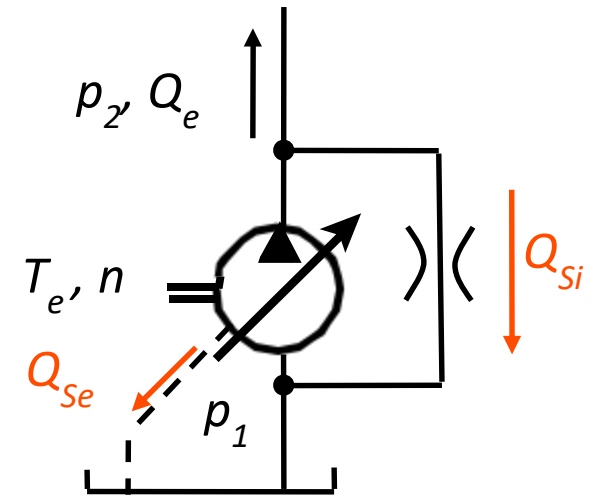
The power required to drive the pump:

$$P = T_e \cdot \omega = T_e \cdot 2 \cdot \pi \cdot n = 636.62 \text{ Nm} \cdot 2 \cdot \pi \cdot \frac{1800}{60} \cdot \text{s}^{-1} = 120 \text{ kW}$$

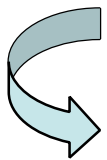
# Μετρήσεις steady state

The aim of steady state measurements is determination of steady state characteristics of pumps.

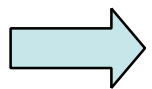
- Losses and their dependency on operating parameters
- Efficiency and its dependency on operating parameters
- Effective torque  $T_e$  and effective volumetric Flow rate  $Q_e$  in the whole parameter range



Under stable conditions, all parameters remain constant, including temperatures !



Parameters to be measured:



Temperatures  $\vartheta_1, \vartheta_2, \vartheta_{se}$

Inlet pressure  $p_1$       Outlet pressure  $p_2$

Torque  $T_e$       Shaft speed  $n$

Volume flow rate at pump outlet  $Q_e = Q_2$

## Συμπεριφορά steady state

$V_i$  derived displacement volume to be determined during measurements

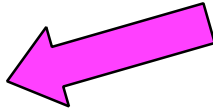
$$Q_e = \alpha \cdot V_{max} \cdot n - Q_s$$



$Q_s$  must be calculated using measurement results

Measured value

$$\Delta p = p_2 - p_1$$



$$T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_s$$



$T_s$  must be calculated using measurement results

$$P_{in} = P_{out} + P_s = \Delta p \cdot Q_e + P_s = T_e \cdot 2 \cdot \pi \cdot n$$



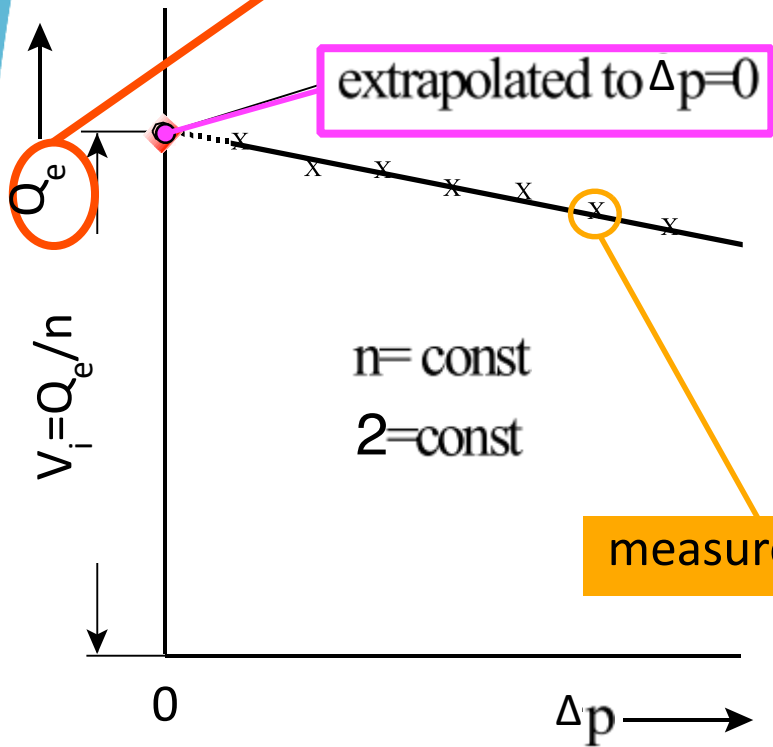
$P_s$  must be calculated using measurement results



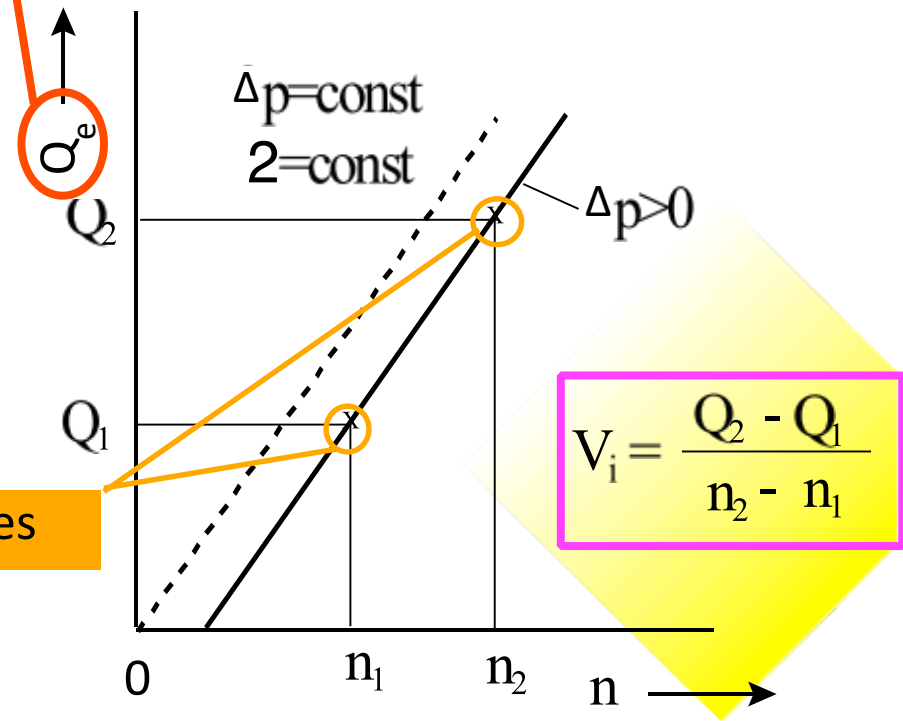
# Εξαγωγή παροχής όγκου

Measurement of effective volume flow rate at pump outlet under defined conditions

Method by Toet



Method defined in ISO 8426

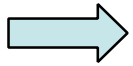


$V_i$  required for determination of losses and volumetric and torque efficiency

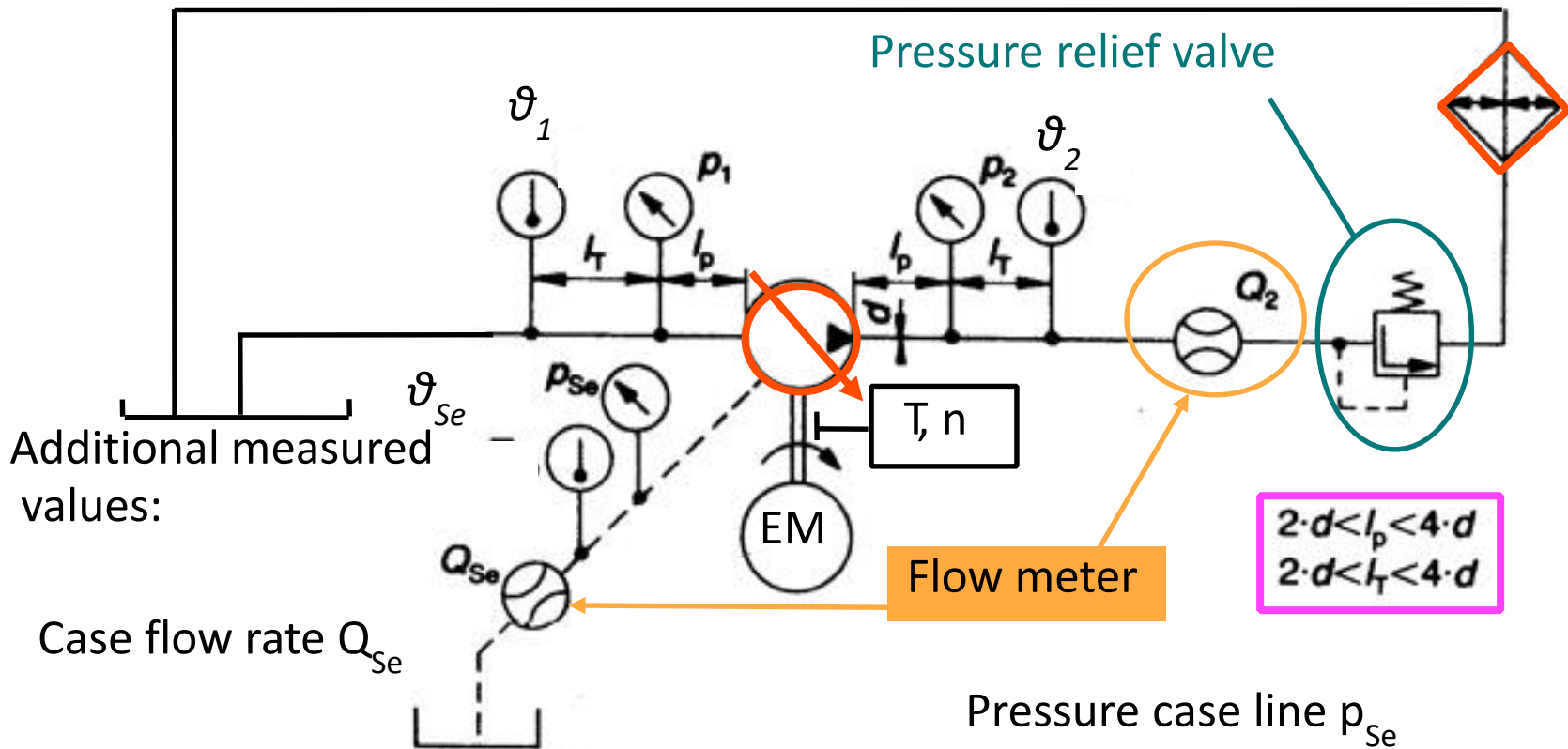
# Κύκλωμα δοκιμής

Measured Values: Inlet pressure  $p_1$  Torque  $T$  Shaft speed  $n$   
 Outlet pressure  $p_2$  Temperature  $\vartheta_1, \vartheta_2, \vartheta_{se}$   
 Volume flow rate at pump outlet  $Q_2$

ISO 4409



Temperature  $\theta_1$  must remain constant during measurements



# Μετρήσεις steady state

In case that  $Q_2$  is measured in low pressure line, the measured value must be corrected with respect to  $p_2$  and  $\theta_2$

$$Q_2 = Q_3 \left[ 1 - \frac{p_2 - p_3}{K} + \beta_\theta (\theta_2 - \theta_3) \right]$$

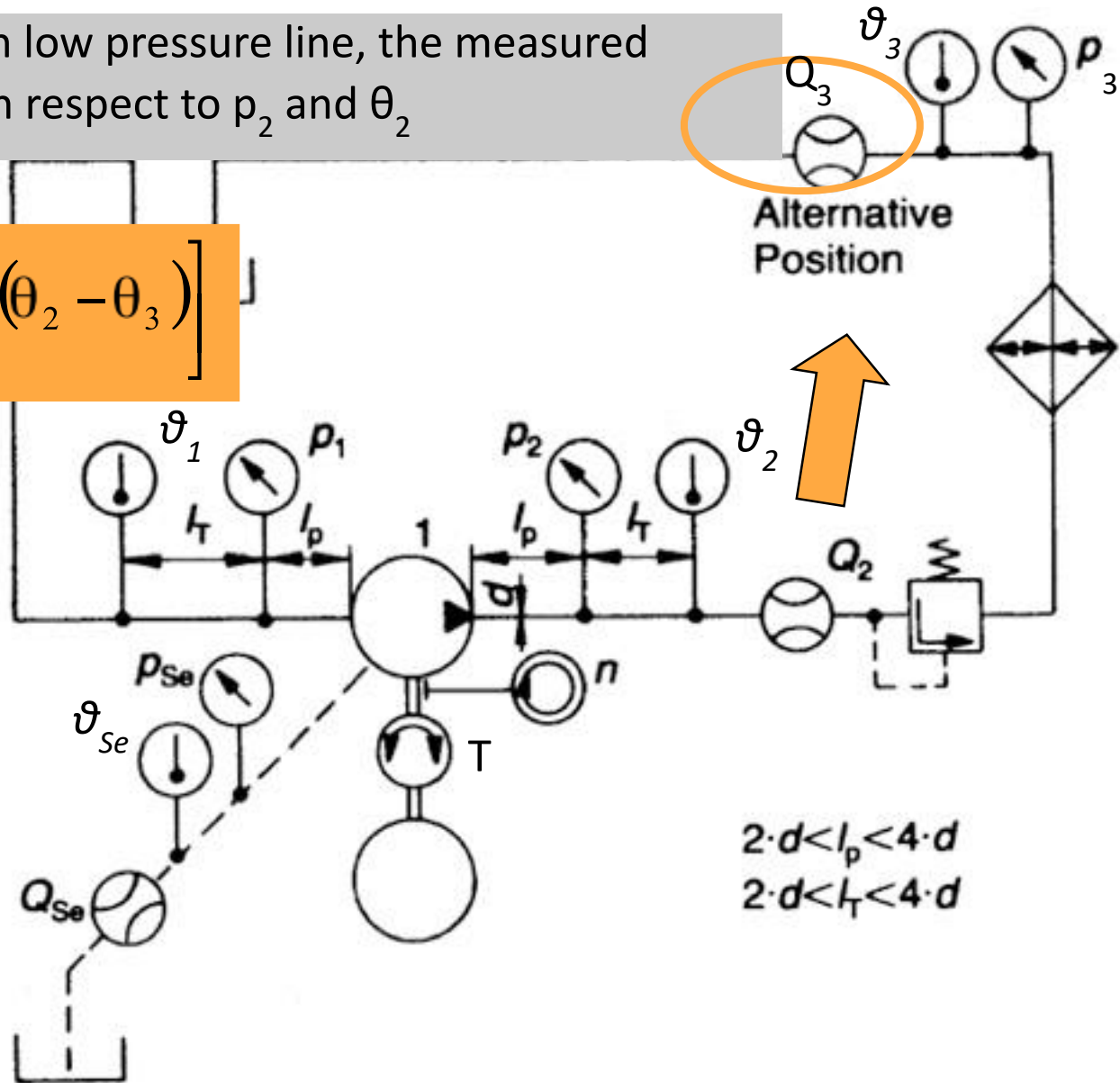
with

$K$  ... bulk modulus

$\beta_\theta$  ... thermal volumetric expansion coefficient

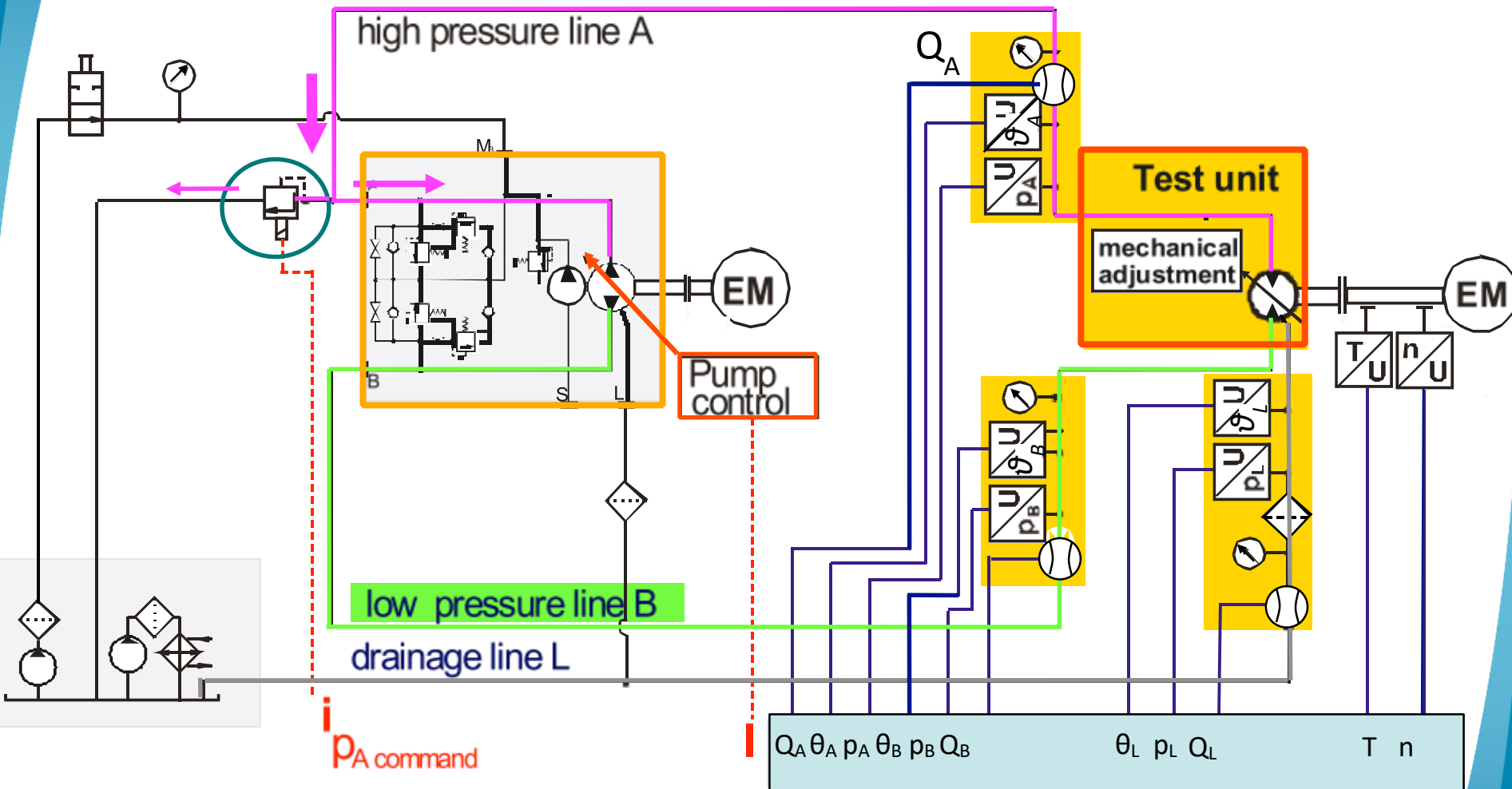
$$K = 2 \cdot 10^9 \text{ Pa}$$

$$\beta_\theta = 0.65 \cdot 10^{-3} \text{ K}^{-1}$$



# Μετρήσεις steady state

Measurement in pumping and motoring mode



Less temperature problems, because only a small amount of volume flow is throttled in pressure relief valve

# Μετρήσεις steady state

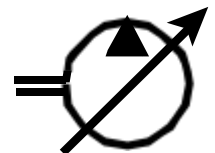
## ISO measurement accuracy classes

Table 1: Permissible systematic errors of measuring instruments as determined during calibration

Parameter of measuring instrument	Permissible systematic errors for classes of measurement accuracy		
	A	B	C
Rotational frequency [%]	± 0.5	± 1.0	± 2.0
Torque [%]	± 0.5	± 1.0	± 2.0
Volume flow rate [%]	± 0.5	± 1.5	± 2.5
Pressure below 2 bar gauge [bar]	± 0.01	± 0.03	± 0.05
Pressure greater than or equal to 2 bar gauge [%]	± 0.5	± 1.5	± 2.5
Temperature [°C]	± 0.5	± 1.0	± 2.0

### Permissible temperature variation

Accuracy class	A	B	C
Temperature variation [K]	±1.0	±2.0	±4.0



# Μετρήσεις πίεσης

The different types of pressure differ only with respect to their reference point.

Types of pressure

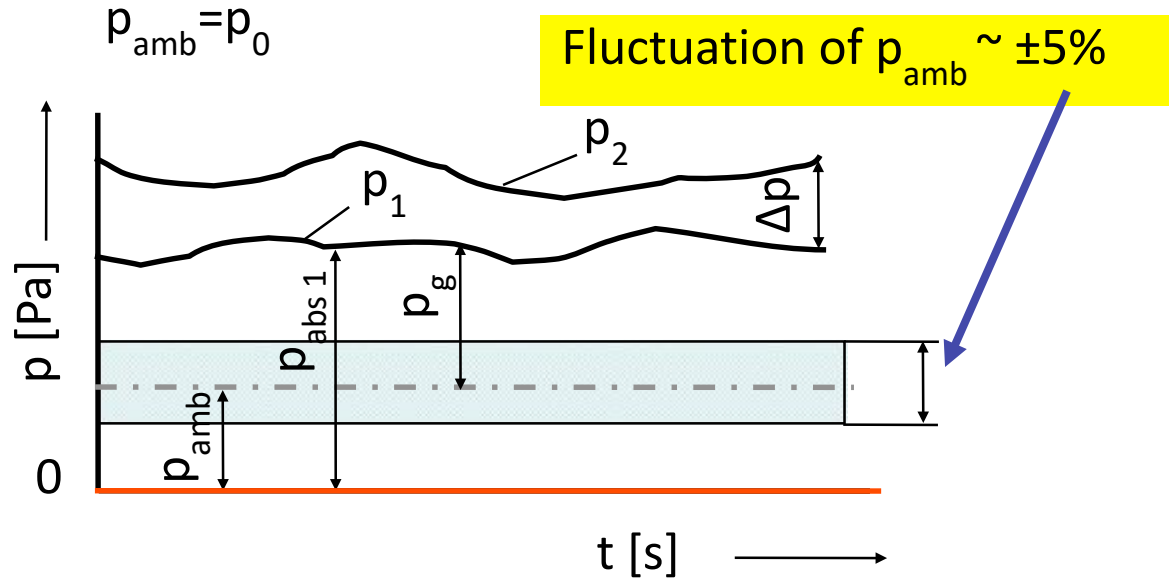
Absolute pressure  $p_{abs}$

Atmospheric pressure  $p_{amb}$

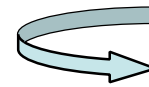
Differential pressure  $\Delta p$

Gauge pressure  $p_g$

$$p_g = p_{measured} - p_{amb}$$

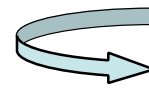


Direct measuring pressure instruments



Using a liquid column

Indirect measuring pressure instruments



Using the effect of a pressure acting on a material or on bodies of a certain shape

Electrical pressure sensors

# Μετρητές πίεσης

## Electrical Pressure Sensors

Inductive  
Principle

Capacitive  
Principle

Resistive  
Principle

Piezoelectric  
Effect

Magnetic Field  
Sensors

LVDT - sensor  
LVR - sensor

Potentiometric  
Sensors

Hall effect

Metallic Strain  
Gauge

Piezoresistive  
Sensors

Strain Foile  
Gauge

Thick-film Strain  
Gauge

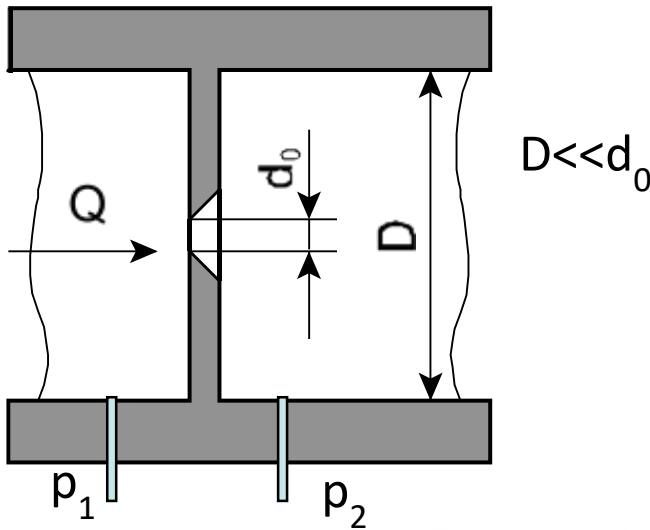
Gauge

Semiconductor  
Materials

# Μετρητές παροχής

Flow measuring instrument is defined as device which measures the flow rate of a fluid.

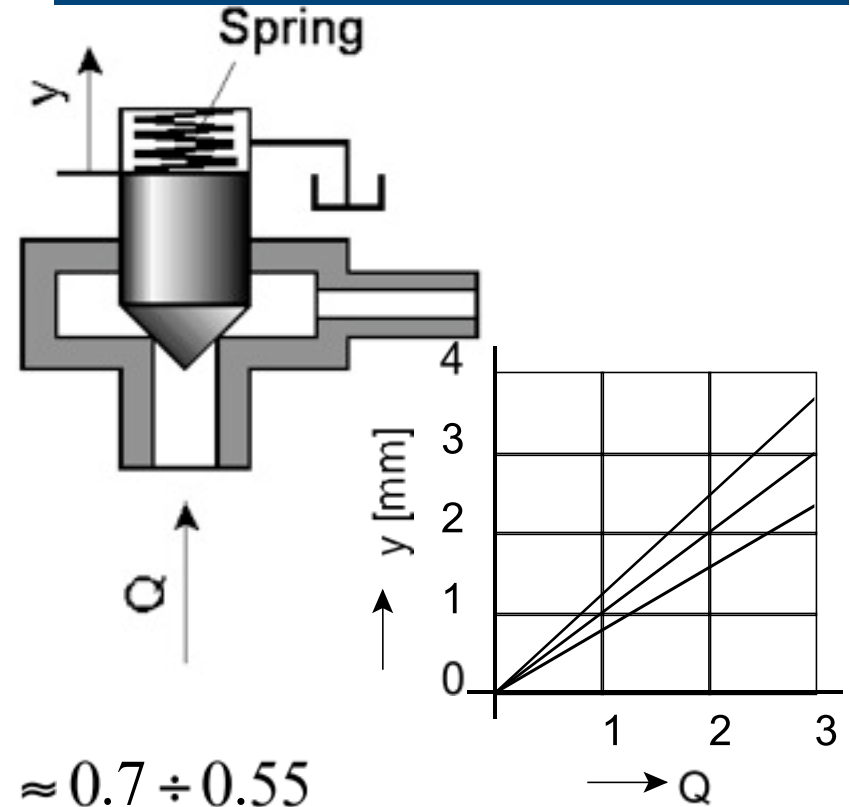
Pressure difference across an orifice



$$Q = \alpha_D \cdot \frac{\pi \cdot d^2}{4} \sqrt{\frac{2 \cdot (p_1 - p_2)}{\rho}}$$

$\alpha_D$ ...flow discharge coefficient

Displacement of a spring loaded floated element



$$\alpha_D \approx 0.7 \div 0.55$$

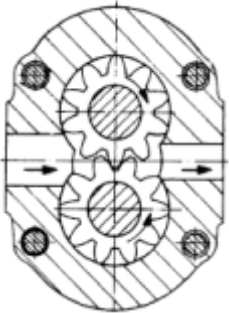
➔ Should not be used for determination of steady state characteristics!



# Μετρητές παροχής

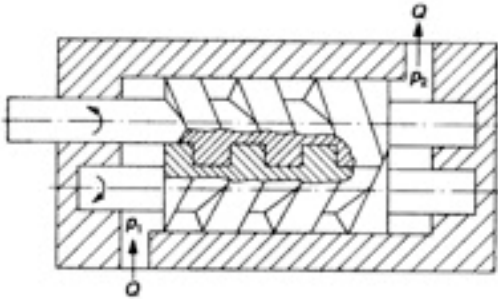
Flow meter – device which directly indicates the rate of flow of a fluid

Displacement principle



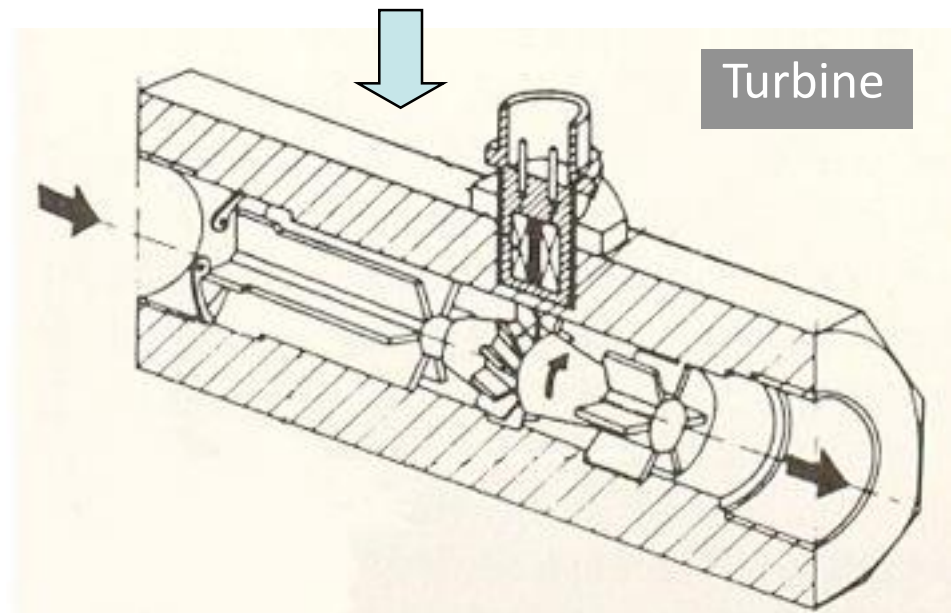
Gear flow meter

Screw flow meter



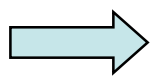
Hydrodynamic principle

Using kinetic energy of a fluid to drive a rotating system of blades (an impeller), whereas the rotational speed of the rotor is measured with an electric speed sensor (frequency measurement device)



# Διαδικασία μέτρησης

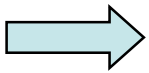
1. Keep  $p_1$  and  $\theta_1$  constant
2. Adjust different pressure levels at constant shaft speed
3. Record all measured values under steady state conditions



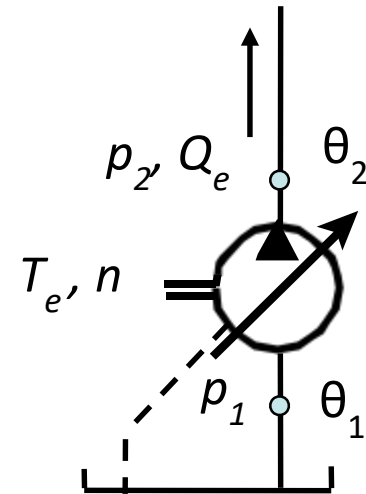
Repeat measurements for different speed settings



In case of variable displacement pump repeat measurements for 75%, 50% and 25% of  $V_{max}$



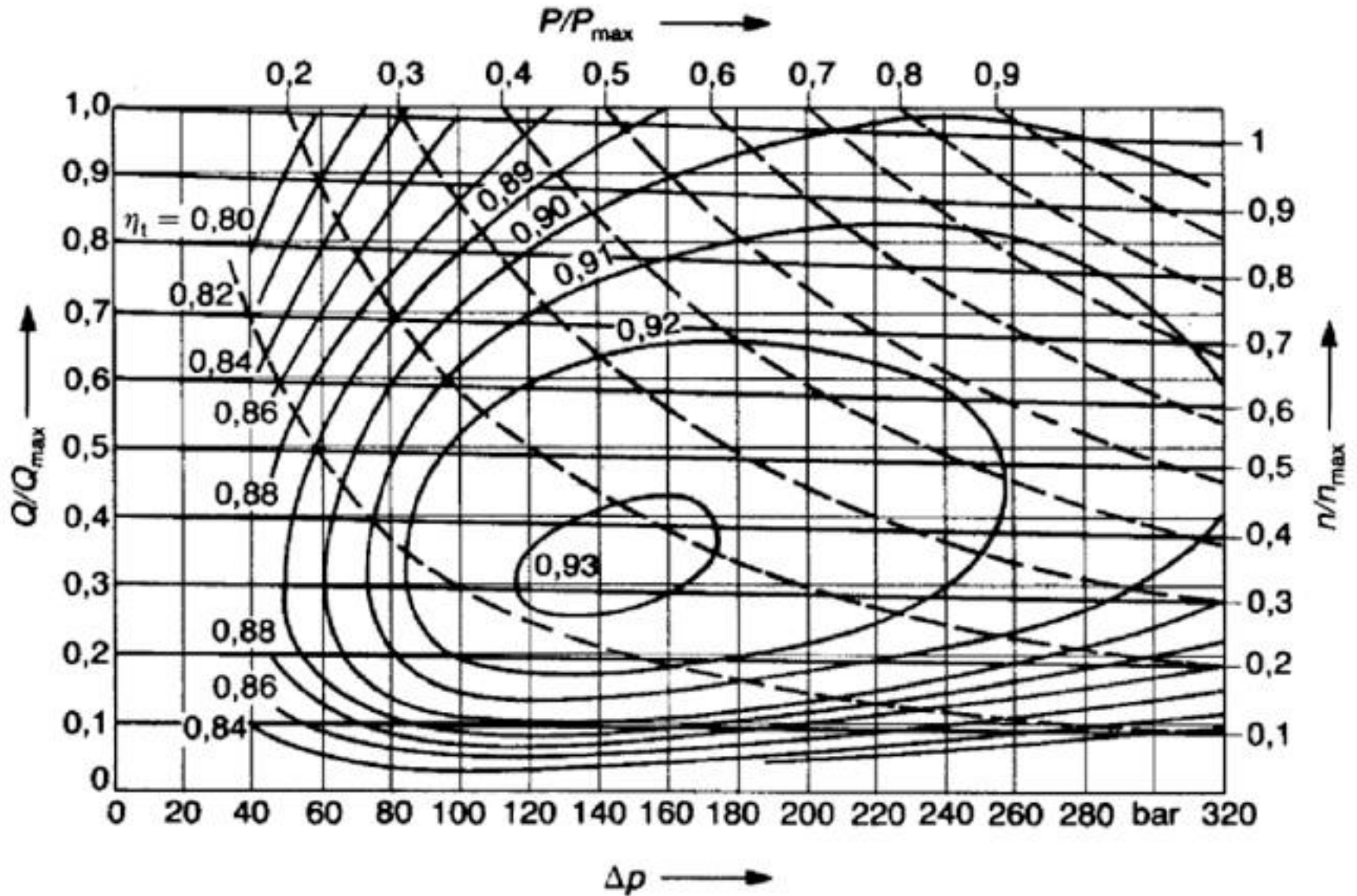
Repeat measurements for different temperatures  $\theta_1$



## Provide table with measurement results

No.	n	T	$p_1$	$\theta_1$	$p_2$	$Q_2$	$\theta_2$	$p_{Se}$	$Q_{Se}$	$\theta_{Se}$
	[RPM]	[Nm]	[bar]	[°C]	[bar]	[l/min]	[°C]	[bar]	[l/min]	[°C]
1	2001.46	15.03	19.77	50.50	25.11	145.13	61.20	1.20	0.21	65.30
2	2001.63	68.78	19.79	50.30	69.65	144.02	60.30	1.25	0.43	66.00
3	2001.79	130.59	19.86	50.30	120.82	142.92	60.40	1.28	0.56	65.20

# Χακτηριστικά steady state



# Ογκομετρικές απώλειες

Effective volume flow rate is reduced due to compressibility of the fluid

$$\int_B^C \frac{dV}{V} = \int_B^C -\frac{1}{K_A} dp \quad \Rightarrow \quad \ln V_C - \ln V_B = -\frac{1}{K_A} (p_C - p_B)$$

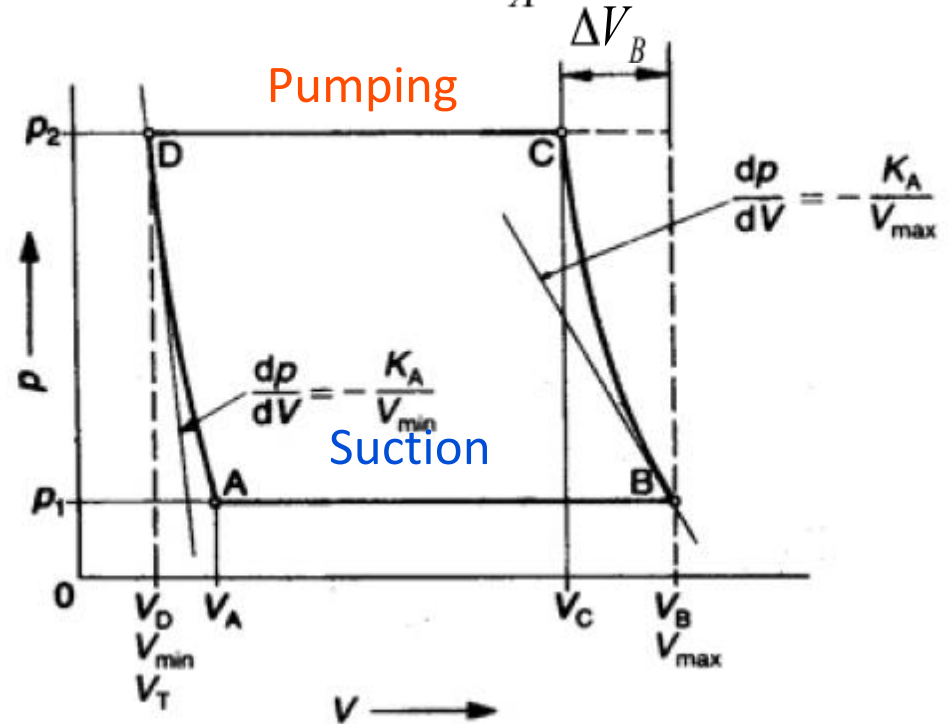
$$\Delta V_B = V_B \left( 1 - e^{-\frac{1}{K_A} (p_C - p_B)} \right)$$

simplified

$$\Delta V_B = V_B \frac{\Delta p}{K_A}$$

$$Q_{SK} = n \Delta V_B$$

with n ... pump speed



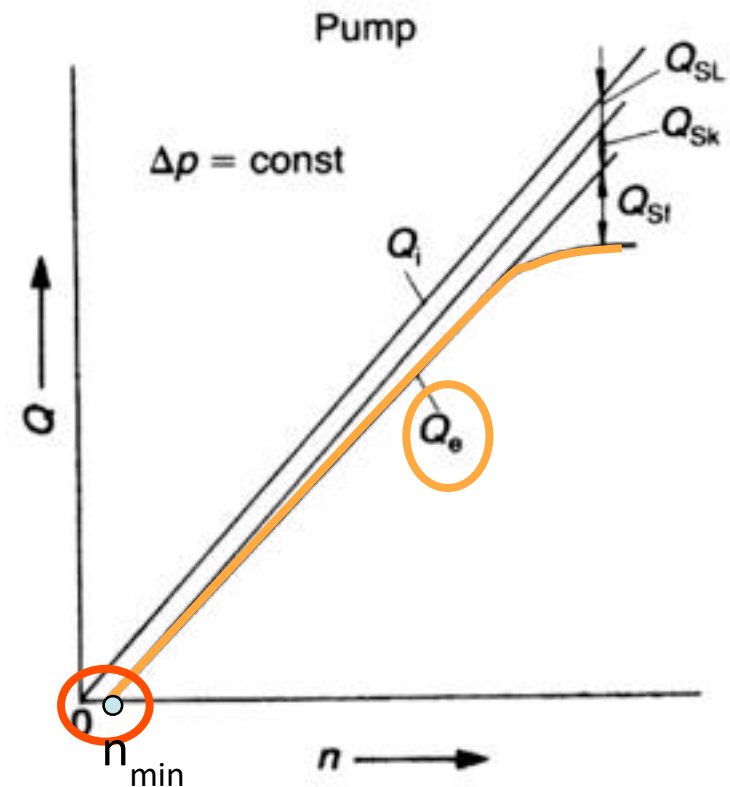
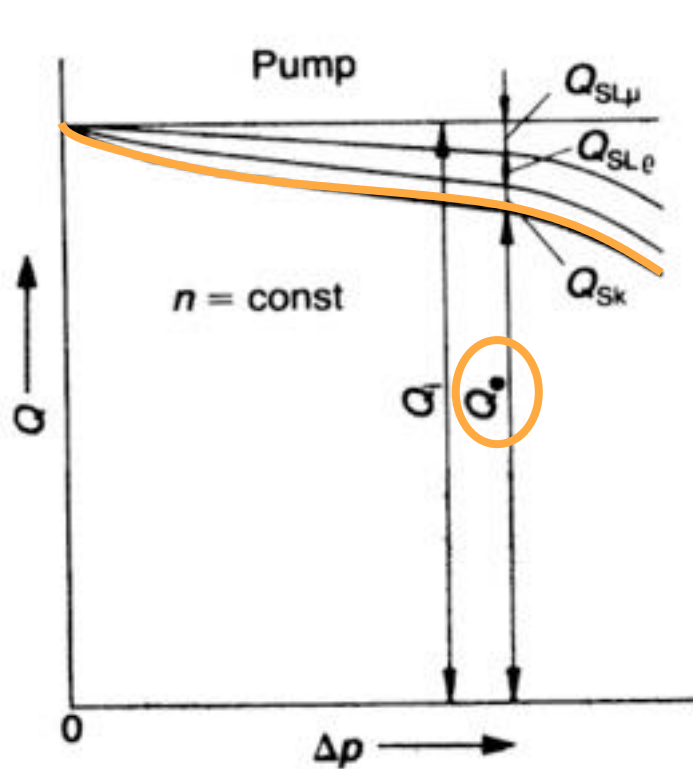
# Συμπεριφορά steady state

of a real displacement machine

$$Q_i = V n = \alpha V_{max} n$$

Effective volumetric flow rate

$$Q_e = Q_i - Q_S$$



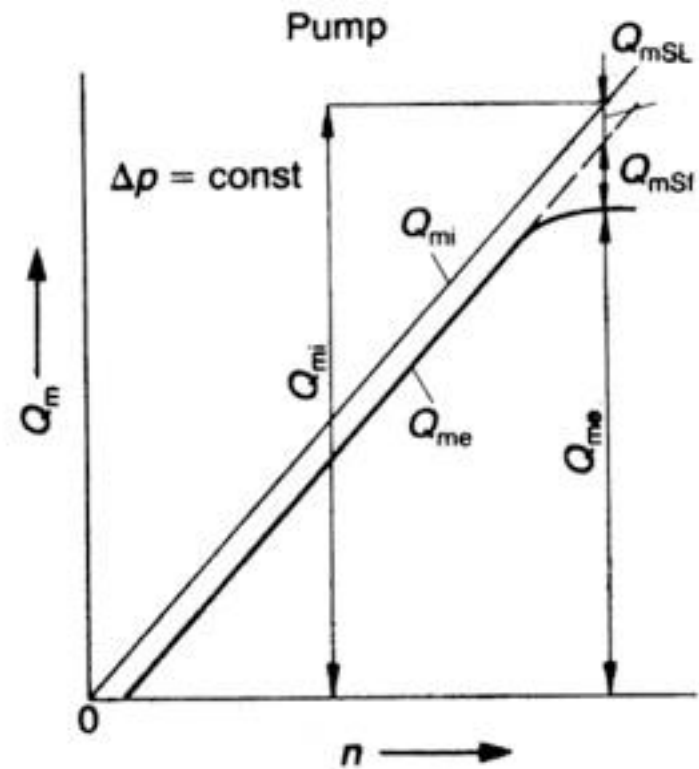
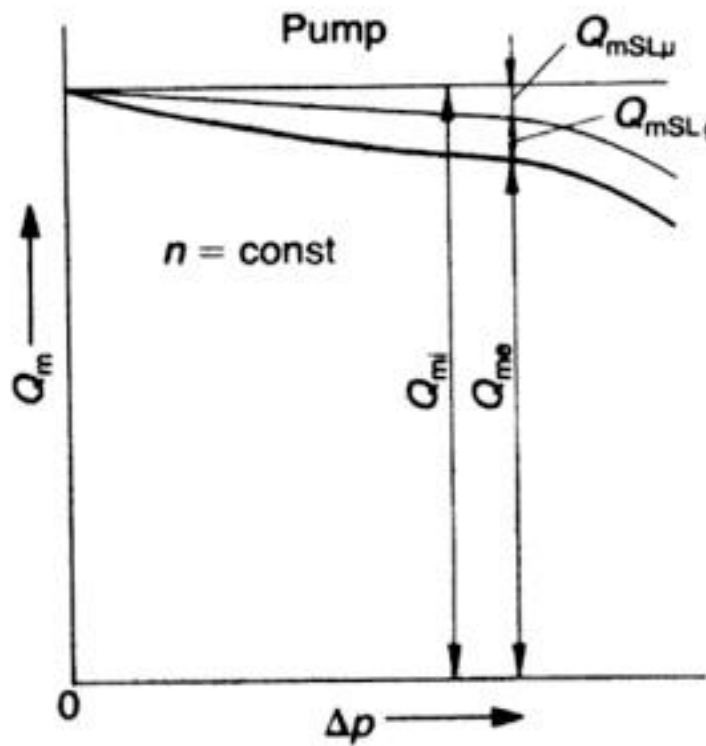
$$Q_S = f(\Delta p, n, V, \theta)$$

$\theta$  ... temperature

# Συμπεριφορά steady state

Effective mass flow at pump outlet  $Q_{me}$

Loss component due to compressibility does not occur!



*Ευχαριστώ για την προσοχή σας*

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