Αρχές λειτουργίας και τεχνολογία μηχανών θετικής μετατόπισης

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24/02/2020
Περιεχόμενα

- Displacement machines – design principles & scaling laws
- Power density comparison between hydrostatic and electric machines
- Volumetric losses, effective flow, flow ripple, flow pulsation
- Steady state characteristics of an ideal and real displacement machine
- Torque losses, torque efficiency
due to compressibility of a real fluid

\[ V_{\text{min}} = V_T \] with \( V_T \) dead volume

\[ K_A. \text{ adiabatic bulk modulus} \]
due to viscosity & compressibility of a real fluid
Pressure drop between displacement chamber and port

Pressure in displacement chamber

Pump

Motor
Electric Motor

\[ I = J \cdot b \cdot h \]

\[ F_e = I \cdot B \cdot L \cdot \sin \alpha \]

\[ B \ldots \text{magnetic flux density [T or Vs/m}^2\text{]} \]

Torque:

\[ T = I \cdot B \cdot L \cdot r \cdot \sin \alpha \]

Hydraulic Motor

\[ F_h = p \cdot L \cdot h \]

\[ T = p \cdot L \cdot h \cdot r \]
\[ P = T \omega = T \cdot 2\pi n \]

For electric motor follows:
\[ P = I B L r \cdot 2\pi n \]
assuming \( \alpha = 90^\circ \)

For hydraulic motor follows:
\[ P = p \cdot L \cdot h \cdot r \cdot 2\pi \cdot n \]

Force density:

Electric Motor
\[ \frac{F_e}{L \cdot h} = \frac{J \cdot b \cdot h \cdot B \cdot L}{L \cdot h} = J \cdot b \cdot B \]
with a cross section area of conductor: \( 9 \times 10^{-6} \text{ m}^2 \)

Hydraulic Motor
\[ \frac{F_h}{L \cdot h} = p \]
up to \( 5 \times 10^7 \text{ Pa} \)
Electric Machine  Positive displacement machine

\[
\frac{\text{mass}}{\text{power}} = 1 \ldots 15 \text{ kg/kW} \quad \text{0.1} \ldots 1 \text{ kg/kW}
\]

- 10 times lighter
- min. 10 times smaller
- much smaller mass moment of inertia (approx. 70 times)
- much better dynamic behavior of displacement machines
Μηχανές θετικής μετατόπισης

**Piston Machines**
- Axial Piston Machines
- In-line Piston Machines
- Radial Piston Machines

**Gear Machines**
- External Gear
- Internal Gear
- Annual Gear

**Vane Machines**

**Screw Machines**

**Swash Plate Machines**

**Bent Axis machines**
- with external piston support
- with internal piston support

**Fixed displacement machines** ↔ **Variable displacement machines**
Piston stroke = f (β,R)

Variable displacement pump
Requires continuous change of β
Torque generation on cylinder block

Swash plate design

Radial force $F_R$ exerted on piston!

Driving flange must cover radial force

Bent axis machines
Αντλίες αξονικών εμβόλων

Openings in cylinder bottom
In case of plane valve plate

In case of spherical valve plate
Connection of displacement chambers with suction and pressure port

Plane valve plate
Inlet opening
Outlet opening

Inlet
Outlet

Αντλίες αξονικών εμβόλων
Kinematic reversal: pump with rotating swash plate

- Suction valve
- Pressure valve for each cylinder

Outlet
Inlet

can only work as pump
Because of check valves
Αντλίες αξονικών εμβόλων-Σύγκριση

- Simple and compact design
- Short response time, high bandwidth
- Through going shaft
- Long service life, low loaded bearings
- Limited swash plate angle $\beta_{\text{max}}$ ca. 21°
- High radial piston forces

- higher max. speed
- Angle $\beta$ up to 45°
- Less losses
- High loaded bearings
- Expensive design
- Synchronisation required
Displacement volume of a variable displacement machine:

\[ V = \alpha \cdot V_{max} \]
The pump size is determined by the displacement volume $V$ [cm$^3$/rev]. Usually a proportional scaling law, conserving geometric similarity, is applied, resulting in stresses remaining constant for all sizes of units.

$$T = \frac{\Delta p \cdot V}{2 \cdot \pi} \quad Q = V \cdot n \quad \Delta p = p_2 - p_1$$

First Order Scaling Laws:

\[ L = \lambda \cdot L_0 \quad \lambda^3 \cdot T_0 \quad \lambda^2 \cdot P_0 \quad \lambda^3 \cdot V_i \quad \lambda^3 \cdot m_0 \quad \lambda^{-1} \cdot n_0 \]

Assuming same maximal operating pressures for all unit sizes and a constant maximal sliding velocity!
The maximal shaft speed of a given pump is 5000 rpm. The displacement volume of this pump is \( V = 40\, \text{cm}^3/\text{rev} \). The maximal working pressure is given with 40 MPa. Using first order scaling laws, determine:
- the maximal shaft speed of a pump with 90 cm\(^3\)/rev
- the torque of this larger pump
- the maximal volume flow rate of this larger pump
- the power of this larger pump

For the linear scaling factor follows:

\[
\lambda = \sqrt[3]{\frac{V}{V_0}} = \sqrt[3]{\frac{90}{40}} = 1.31
\]

Maximal shaft speed of the larger pump:

\[
n = \lambda^{-1} \cdot n_0 = 1.31^{-1} \cdot 5000\, \text{rpm} = 3816.8\, \text{rpm}
\]

Torque of the larger pump:

\[
T = \frac{\Delta p \cdot V}{2 \cdot \pi} = \frac{40 \cdot 10^6\, \text{Pa} \cdot 90 \cdot 10^{-6}\, \text{m}^3}{2 \cdot \pi} = 573.25\, \text{Nm}
\]

Maximal volume flow rate:

\[
Q_{\text{max}} = V \cdot n_{\text{max}} = 90 \cdot 10^{-6}\, \text{m}^3/\text{rev} \cdot 3816.8\, \text{rpm} = 0.3435\, \text{m}^3/\text{min} = 343.51/\text{min}
\]

Power of the larger pump:

\[
P = \Delta p \cdot Q = 40 \cdot 10^6\, \text{Pa} \cdot 0.3435\, \text{m}^3 \cdot \frac{1}{60} = 229\, \text{kW}
\]
Volumetric Losses

Torque Losses

Axial piston machine – swash plate design

Απώλειες στις αντλίες θετικής μετατόπισης
Effective Flow rate:

\[ Q_e = \alpha V_{\text{max}} n - Q_s \]

Effective torque:

\[ T_e = \frac{\Delta p \cdot \alpha \cdot V_{\text{max}}}{2 \cdot \pi} + T_s \]

\[ Q_{se} \ldots \text{external volumetric losses} \]

\[ Q_{si} \ldots \text{internal volumetric losses} \]

\[ \Delta p = p_2 - p_1 \]

\[ T_s \ldots \text{torque losses} \]
\[
Q_S = \sum_{i=1}^{n} Q_{Sei} + \sum_{j=1}^{m} Q_{Si_j} + Q_{SK} + Q_{Sf}
\]

\(Q_{SL}\) external and internal volumetric losses = flow through laminar resistances:

\[
Q_{SL} = C_\mu \cdot \frac{\Delta p}{\mu}
\]

\[
Q = \frac{b \cdot h^3 \cdot \Delta p}{12 \cdot \mu}
\]

Assuming const. gap height

\[
\mu = f(\theta, p)
\]
Instantaneous volumetric flow $Q_a$

$$Q_a = \frac{dV}{dt} = f(\varphi)$$

Volumetric flow displaced by a displacement chamber

The instantaneous volumetric flow is given by the sum of instantaneous flows $Q_{ai}$ of each displacement element:

$$Q_a = \sum_{i=1}^{k} Q_{ai}$$

$k$ ... number of displacement chambers, decreasing their volume, i.e. being in the delivery stroke

$z$ is an even number $k = \frac{z}{2}$

$z$ is an odd number $k = \frac{z}{2} + 0.5$ or $k = \frac{z}{2} - 0.5$

Flow pulsation of pumps

Pressure pulsation
Non-uniformity grade of volumetric flow is defined:

\[ \delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{mi}} \]

\[ Q_{mi} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2} \]

\[ \delta_Q = 2 \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{\text{max}} + Q_{\text{min}}} \]
Torque loss due to viscous friction in gaps (laminar flow)

\[ T_{S\mu} = k_{T\mu} \cdot \frac{\mu}{h} \cdot n = C_{T\mu} \cdot \mu \cdot n \]

\( h \ldots \) gap height

Torque loss to overcome pressure drop caused in turbulent resistances

\[ T_{Sp} = C_{Tp} \cdot \rho \cdot n^2 \]

Torque loss linear dependent on pressure

\[ T_{Sp} = C_{Tp} \cdot \Delta p \]

\( \Delta p_s = \lambda \cdot \frac{l}{d} \cdot \rho \cdot \frac{v^2}{2} + \xi \cdot \rho \cdot \frac{v^2}{2} \)

\( \xi \ldots \) drag coefficient

\( \lambda \ldots \) flow resistance coefficient

\( \lambda_{turbulent} = \frac{0.3164}{\sqrt[4]{Re}} \)

Effective torque required at pump shaft

\[ T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_s \]
Συμπεριφορά steady state

Torque losses of a real displacement machine

\[ T_S = f(n, \Delta p, V, \theta) \]
Effective torque $T_e$

$$T_e = T_i + T_s = \frac{\Delta p \cdot \alpha \cdot V_{\text{max}}}{2 \cdot \pi} + T_s$$

Effective torque $T_e$

$\Sigmaυμπεριφορά \, \text{steady state}$

$T_S = f(\Delta p, n, V, \theta)$
Piston displacement: \( s_p = -z \)

\[ s_p = -R \cdot \tan \beta \cdot (1 - \cos \varphi) \]

Piston stroke:

\[ H_p = 2 \cdot R \cdot \tan \beta \]

R ... pitch radius

Outer dead point AT

Inner dead point IT

\[ z = b \cdot \tan \beta \]

\[ b = R - y \]

\[ y = R \cdot \cos \varphi \]
Piston velocity in z-direction:

\[ v_P = \frac{ds_P}{dt} = \frac{ds_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega \cdot R \cdot \tan \beta \cdot \sin \varphi \]

Piston acceleration in z-direction:

\[ a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\varphi} \cdot \frac{d\varphi}{dt} = -\omega^2 \cdot R \cdot \tan \beta \cdot \cos \varphi \]

Circumferential speed

\[ v_u = R \cdot \omega \]

Centrifugal acceleration:

\[ a_u = R \cdot \omega^2 \]

Coriolis acceleration \( a_c \) is just zero, as the vector of angular velocity \( \omega \) and the piston velocity \( v_P \) run parallel.
Geometric displacement volume:
\[ V_g = z \cdot A_p \cdot H_p \]
\( z \) ... number of pistons

In case of pistons arranged parallel to shaft axis:

\[ V_g = z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta \]

Geometric flow rate:
\[ Q_g = n \cdot z \cdot \frac{\pi \cdot d_p^2}{2} \cdot R \cdot \tan \beta \]

For an ideal pump without losses

Mean value over time

Instantaneous volumetric flow:
\[ Q_a = \sum_{i=1}^{k} Q_{ai} \]
\( k \) ... number of pistons, which are in the delivery stroke

with \( Q_{ai} \) ... instantaneous volumetric flow of individual piston

\[ v_p = \omega \cdot R \cdot \tan \beta \cdot \sin \varphi \]

\[ Q_{ai} = v_p \cdot A_p = \omega \cdot A_p \cdot R \cdot \tan \beta \cdot \sin \varphi_i \]
In case of **even** number of pistons: \( k = 0.5 \cdot z \)

In case of **odd** number of pistons:

\[
k_1 = \frac{z}{2} + 0.5 \quad \text{for} \quad 0 < \varphi \leq \frac{\pi}{z}
\]

and \( k_2 = \frac{z}{2} - 0.5 \quad \text{for} \quad \frac{\pi}{z} < \varphi \leq 2 \cdot \frac{\pi}{z} \)

\[
Q_a = \sum_{i=1}^{k} Q_{ai}
\]
kinematic flow and torque pulsation due to a finite number of piston

Flow Pulsation:  

Non-uniformity grade:

$$\delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{mi}}$$  with  $$Q_{mi} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2}$$

Even number of pistons:

$$\delta_Q = \frac{\pi}{2z} \tan \left( \frac{\pi}{2z} \right)$$

Odd number of pistons:

$$\delta_Q = \frac{\pi}{4z} \tan \left( \frac{\pi}{4z} \right)$$

Torque Pulsation

$$\delta_T = \frac{T_{\text{max}} - T_{\text{min}}}{T_{mi}}$$  with  $$T_{mi} = \frac{T_{\text{max}} + T_{\text{min}}}{2}$$
kinematic flow and torque pulsation due to a finite number of piston

Non-uniformity

Even number of pistons: \( f = z \cdot n \)

Odd number of pistons: \( f = 2 \cdot z \cdot n \)
Non-uniformity grade:

\[ \delta_Q = \frac{Q_{\text{max}} - Q_{\text{min}}}{Q_{mi}} \quad \text{with} \quad Q_{mi} = \frac{Q_{\text{max}} + Q_{\text{min}}}{2} \]

Kinematic non-uniformity grade for piston machines:

<table>
<thead>
<tr>
<th>Number of pistons z</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-uniformity grade (\delta)</td>
<td>0.140</td>
<td>0.325</td>
<td>0.049</td>
<td>0.140</td>
<td>0.025</td>
<td>0.078</td>
<td>0.015</td>
<td>0.049</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Volumetric losses \(Q_s = f(\phi)\) and

\[ Q_s = f(\Delta p, n, V_i, \theta) \]

Flow pulsation of a real displacement machine is much larger than the flow pulsation given by the kinematics.
Flow pulsation leads to pressure pulsation at pump outlet
Piston Machines

Axial Piston Machines

In-line Piston Machines

Radial Piston Machines

Gear Machines

External Gear

Internal Gear

Annual Gear

Vane Machines

Swash Plate Machines

Bent Axis machines

with external piston support

with internal piston support

Screw Machines

Variable displacement machines

Others

Fixed displacement machines
Axial gaps between housing and the gear pair must be very small to seal the displacement chamber.

Radial gaps between teeth addendum circle and housing.

\[ Q_e = V \cdot n - Q_s \]
Expression for the theoretical flow rate of an external gear pump

\[ D_o = \text{the outside diameter of gear teeth} \]
\[ D_i = \text{the inside diameter of gear teeth} \]
\[ L = \text{the width of gear teeth} \]
\[ N = \text{the speed of pump in RPM} \]
\[ V_D = \text{the displacement of pump in m/rev} \]
\[ M = \text{module of gear} \]
\[ z = \text{number of gear teeth} \]
\[ \alpha = \text{pressure angle} \]

Volume displacement is

\[ V_D = \frac{\pi}{4} (D_o^2 - D_i^2) L \]
\[ D_i = D_o - 2(\text{Addendum} + \text{Dendendum}) \]

Theoretical discharge is

\[ Q_T (m^3/min) = V_D (m^3/\text{rev}) \times N (\text{rev/min}) \]

If the gear is specified by its module and number of teeth, then the theoretical discharge can be found by

\[ Q_T = 2\pi L m^2 N \left[ z + \left( 1 + \frac{\pi^2 \cos^2 20}{12} \right) \right] m^3/\text{min} \]
Expression for the theoretical flow rate of an external gear pump
Outlet 1 and inlet 2 can be connected

\[ p_2 = p_3 \]

or the pump can have two separate outlets

\[ p_1 = p_3 \]

\[ \Delta p_1 \approx \Delta p_2 \]

The driving gear is pressure balanced!
Using teeth of standard involute design requires a combination where the pinion has two or more fewer teeth than the ring gear! Pinion and ring gear are then separated by a crescent shaped divider.

Advantages:
- Better suction ability
- Higher efficiency
- More compact design
- Less noise emission

Longer duration of teeth meshing leads to better sealing function.
Many different tooth profiles have been applied in the recent past.

Crescent shaped divider

Γραναζωτή αντλία εσωτερικής οδόντωσης
Applying specially generated tooth curves it can be achieved, that the inner rotor (the pinion) has only one tooth less than the ring gear, thus eliminating the crescent-shaped divider.

Each tooth of the pinion maintains continuous sliding contact with a tooth of the ring gear, providing fluid tight engagement.

Relative sliding velocity between pinion and ring gear is very small for quiet operation and long service life.

\[
\frac{n_2}{n_1} = \frac{z_1}{z_2} = \frac{z_1}{z_1 + 1} = n_1 \cdot \left(1 - \frac{1}{z_2}\right) = n_1 \cdot \left(1 + \frac{1}{z_1}\right)^{-1}
\]
Ring gear ($z_2$) fixed

Rotating pinion ($z_1$)

Displacement volume is given by $z_1$ times $z_2$ tooth spaces

Multiple delivery of each tooth space

1 Suction port

2 Pressure port

Outlet

Inlet

$z_2 = z_1 + 1$
Only one direction of shaft rotation possible!
\( A = (1.1 \div 1.3) \cdot \frac{2 \cdot F_z}{p_2} \)

- Driving gear
- Axial gap – pressure compensated
- Pressurized area
- Sealing
- Sliding bearings
- Driven gear
- Improved volumetric efficiency
Radial gap compensation

Bearing bushing

Small pressure zone achievable

Axial gap compensation

Pressurized area

Ακτινικά διάκενα
Παράδειγμα σχεδιασμού

Driving gear
Shaft seal
Driven gear
Bearing bushing performing a radial and axial gap compensation

inlet
outlet
Internal gear pump with axial and radial gap compensation

- Ring gear
- Pinion
- special shaped divider
- Radial gap compensation
- Moveable bearing shell
- Pressurized area
- inlet
- outlet
Mηχανές θετικής μετατόπισης

**Piston Machines**
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**Gear Machines**
- External Gear
- Internal Gear
- Annual Gear

**Vane Machines**

**Screw Machines**

**Swash Plate Machines**

**Bent Axis machines**

**Fixed displacement machines**

**Variable displacement machines**
Synchronization of cylinder block

Cylinder block

Using a universal joint

Driving flange

Using a bevel gear

Αντλίες αξονικών εμβόλων bent axis
Αντλίες αξονικών εμβόλων bent axis

Synchronization of cylinder block

Cardan joint

connecting rod

piston

Piston rod

Synchronization by piston rod

Synchronization by pistons
Αντλίες αξονικών εμβόλων bent axis-κινηματικά

Assuming a fixed connection between link 2 and link 4, achieved by synchronization mechanism has finally two degrees of freedom.

Four link 3D mechanism

Frame (1)
Cylinder (4)
Piston (3)
Driving flange (2)

Piston can rotate about $z_3$-axis

Driving flange (2)

Five link 3D mechanism

Frame (1)
Piston rod (5)
Cylinder (4)
Piston (3)
Driving flange (2)

Piston can rotate about $z_3$-axis and piston rod can rotate about $z_5$-axis
Σχεδιασμός εμβόλου

Short piston with piston rod

Long piston with piston rod

Synchronization by universal joint or bevel gear

Synchronization by pistons or piston rods

Spherical piston with piston ring

Conical piston with piston rings
Driving flange bearings

Pump control device

Spherical valve plate
Παράδειγμα σχεδιασμού

- Driving flange bearings
- Conical piston
- Fixed displacement pump
- Spherical valve plate
with external piston support
Rotating cylinder body

with internal piston support
Stationary cylinder body

Suction

Stroke ring

eccentricity

Suction

Rotating cam or crankshaft

Delivery

Displacement volume adjustable by changing eccentricity e
Radial Piston Pumps

Multiple stroke radial piston pumps

with external piston support

Rotating cylinder body

Stationary stroke ring

Rotating cam

with internal piston support

Stationary cylinder body

Only fixed displacement pumps realizable!
Εξωτερική υποστήριξη εμβόλων

Plane valve plate

Stroke ring

Rotating cylinder body

Piston

Piston rotation enforced by friction force $F_f$
Piston support on outer stroke ring

Stroke ring

Piston roller guide

Stroke ring borne in roller bearings

Piston sliding bearing
Εξωτερική υποστήριξη εμβόλων

- Slipper support
- Stroke ring
- Hydrostatically balanced slipper
- Slipper pocket
- Hydrodynamically balanced slipper
- Ball joint inside the piston
Using a sliding carriage supported using line contact

Stroke ring mounted on a pivot

Change of eccentricity by pivoting the stroke ring about pivot axis
Pump control system
Piston
Control journal
Slipper
Stroke ring

Παράδειγμα σχεδιασμού
With two meshing screws
With three meshing screws

Kοχλιωτή αντλία

Outlet

Inlet

Thread pitch
Κοχλιωτή αντλία

- Σελίδα 14 διάλεξη 7
Unbalanced vane pump

Stator

Rotor

Balanced vane pump

Fixed and variable pump design

Only fixed displacement pump

Πτερυγιοφόρος αντλία
Single stroke vane pump – variable displacement volume

Overcenter pump – the direction of flow can be reversed by change of eccentricity, i.e. without changing the direction of rotation of the drive shaft.

Relatively high friction between axial moveable vanes and rotor & between vanes and stator.

Large radial forces exerted on the rotor.

Limitation of max. operating pressure (20 MPa)
Multiple stroke vane pump

Rigid vane pump

Rotor
External fluid distribution

Internal fluid distribution

Πτερυγιοφόρος αντλία

External fluid distribution

outlet

Internal fluid distribution

Distributor - fixed control journal

inlet

stator

rotor
Πτερυγιοφόρος αντλία

Displacement volume:

\[ V_x = 2 \cdot \pi \cdot \frac{(D^2 - d^2)}{4} \cdot \frac{180 - \alpha}{180} \cdot b - 2 \cdot \left( \frac{D}{2} - \frac{d}{2}, \right) \cdot c \]

Pulsation free flow
Piston Machines

Axial Piston Machines

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with external piston support

with internal piston support

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Vane Machines

Screw Machines

Fixed displacement machines

Variable displacement machines

Mηχανές θετικής μετατόπισης

Annual Gear
Axial piston machine – swash plate design

Volumetric Losses

Torque Losses
Volumetric efficiency: \[ \eta_v = \frac{Q_e}{n \cdot V_i} \]

where \( V_i \) represents the derived displacement volume.

Torque efficiency (hydraulic-mechanical efficiency):
\[ \eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e} \]

Total efficiency:
\[ \eta_t = \frac{P_{out}}{P_{in}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm} \]

The derived displacement volume can only be determined by measurement.
Volumetric efficiency

\[ \eta_v = f(\Delta p, n, V_i, \mu) \]

\[ \eta_v = \frac{Q_e}{n \cdot V_i} \]

\[ Q_e = \alpha \cdot V_{\text{max}} \cdot n - Q_s \]

Dynamic viscosity of fluid:

\[ \mu = f(\theta, p) \] [Pa·s]
Volumetric efficiency

\[ \eta_v = f(\Delta p, n, V_i, \mu) \]

\[ Q_e = \alpha \cdot V_{\text{max}} \cdot n - Q_s \]

Variable displacement pump

Typical values of dynamic viscosity used in displacement machines:

0.0435 Pa \cdot s \div 0.0087 Pa \cdot s

with: Kinematic viscosity \( \nu \) [ cSt, mm\(^2\)/s]

\[ \nu = 10 \text{mm}^2 \cdot \text{s}^{-1} \div 50 \text{mm}^2 \cdot \text{s}^{-1} \text{ with } \rho = 870 \text{kg} \cdot \text{m}^{-3} \]
Απόδοση ροπής

\[ \eta_{hm} = f(\Delta p, n, V_i, \mu) \]

\[ \eta_{hm} = \frac{T_i}{T_e} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot T_e} = 1 - \frac{T_s}{T_e} \]

\[ T_e = \frac{\Delta p \cdot \alpha \cdot V_{\text{max}}}{2 \cdot \pi} + T_s \]

\[ T_s = T_{S\mu} + T_{S\rho} + T_{Sp} + T_{Sc} = C_{\mu} \cdot \mu \cdot n + C_{\rho} \cdot \rho \cdot n^2 + C_p \cdot \Delta p + T_{Sc} \]
Ολικός βαθμός απόδοσης

\[ \eta_t = f(\Delta p, n, V_i, \mu) \]

\[ \eta_t = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{Q_e \cdot \Delta p}{T_e \cdot \omega} = \eta_v \cdot \eta_{hm} \]
Comparison of Efficiencies of Axial Piston Pumps

\[ n = 2000 \text{ rpm} \quad \nu = 20 \text{cSt} \]

- BENT AXIS DESIGN
- SWASH PLATE DESIGN
The displacement pump with a displacement volume of $V = 100 \text{ cm}^3/\text{rev}$ is driven by an electric motor at 1800 rpm. At steady state conditions the pressure difference across the pump is $\Delta p = 380 \text{ bar}$. Determine the effective flow rate at the pump outlet $Q_e$ and the power required to drive this pump. Assume the following values for efficiency:
- volumetric efficiency $\eta_v = 0.87$
- torque efficiency $\eta_{hm} = 0.95$

The effective volume flow rate:

$$Q_e = V \cdot n \cdot \eta_v = 100 \cdot 10^{-6} \text{ m}^3 \cdot 1800 \cdot \frac{1}{60} \cdot s^{-1} \cdot 0.87 = 0.00261 \text{ m}^3 \cdot s^{-1} = 156.6 \text{ l/min}$$

The effective torque yields:

$$T_e = \frac{T_i}{\eta_{hm}} = \frac{\Delta p \cdot V_i}{2 \cdot \pi \cdot \eta_{hm}} = \frac{380 \cdot 10^5 \text{ Pa} \cdot 100 \cdot 10^{-6} \text{ m}^3}{2 \cdot \pi \cdot 0.95} = 636.62 \text{ Nm}$$

The power required to drive the pump:

$$P = T_e \cdot \omega = T_e \cdot 2 \cdot \pi \cdot n = 636.62 \text{ Nm} \cdot 2 \cdot \pi \cdot \frac{1800}{60} \cdot s^{-1} = 120 \text{ kW}$$
The aim of steady state measurements is determination of steady state characteristics of pumps.

- Losses and their dependency on operating parameters
- Efficiency and its dependency on operating parameters
- Effective torque \( T_e \) and effective volumetric flow rate \( Q_e \) in the whole parameter range

Parameters to be measured:

- Inlet pressure \( p_1 \)
- Outlet pressure \( p_2 \)
- Torque \( T_e \)
- Shaft speed \( n \)
- Volume flow rate at pump outlet \( Q_e = Q_2 \)

Under stable conditions, all parameters remain constant, including temperatures!
$V_i$ derived displacement volume to be determined during measurements

$Q_e = \alpha \cdot V_{max} \cdot n - Q_s$\hfill\rightarrow\hfill Q_s$ must be calculated using measurement results

$\Delta p = p_2 - p_1$\hfill\rightarrow\hfill T_s$ must be calculated using measurement results

$T_e = \frac{\Delta p \cdot \alpha \cdot V_{max}}{2 \cdot \pi} + T_s$

$P_{in} = P_{out} + P_s = \Delta p \cdot Q_e + P_s = T_e \cdot 2 \cdot \pi \cdot n$\hfill\rightarrow\hfill P_s$ must be calculated using measurement results
Measurement of effective volume flow rate at pump outlet under defined conditions

Method by Toet

Method defined in ISO 8426

\[ V_i = \frac{Q_2 - Q_1}{n_2 - n_1} \]

\( V_i \) required for determination of losses and volumetric and torque efficiency
**Κύκλωμα δοκιμής**

**Measured Values:**
- Inlet pressure \( p_1 \)
- Torque \( T \)
- Shaft speed \( n \)
- Outlet pressure \( p_2 \)
- Temperature \( \theta_1, \theta_2, \theta_{Se} \)
- Volume flow rate at pump outlet \( Q_2 \)

**Additional measured values:**
- Case flow rate \( Q_{Se} \)
- Pressure case line \( p_{Se} \)

Temperature \( \theta_1 \) must remain constant during measurements.

ISO 4409
In case that $Q_2$ is measured in low pressure line, the measured value must be corrected with respect to $p_2$ and $\theta_2$.

$$Q_2 = Q_3 \left[ 1 - \frac{p_2 - p_3}{K} + \beta_\theta (\theta_2 - \theta_3) \right]$$

with

$K$ ... bulk modulus

$\beta_\theta$ ... thermal volumetric expansion coefficient

$K = 2 \cdot 10^9 \text{ Pa}$

$\beta_\theta = 0.65 \cdot 10^{-3} \text{ K}^{-1}$
Measurement in pumping and motoring mode

Less temperature problems, because only a small amount of volume flow is throttled in pressure relief valve
### ISO measurement accuracy classes

Table 1: Permissible systematic errors of measuring instruments as determined during calibration

<table>
<thead>
<tr>
<th>Parameter of measuring instrument</th>
<th>Permissible systematic errors for classes of measurement accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Rotational frequency [%]</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Torque [%]</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Volume flow rate [%]</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Pressure below 2 bar gauge [bar]</td>
<td>± 0.01</td>
</tr>
<tr>
<td>Pressure greater than or equal to 2 bar gauge [%]</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Temperature [°C]</td>
<td>± 0.5</td>
</tr>
</tbody>
</table>

**Permissible temperature variation**

<table>
<thead>
<tr>
<th>Accuracy class</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature variation [K]</td>
<td>±1.0</td>
<td>±2.0</td>
<td>±4.0</td>
</tr>
</tbody>
</table>
Types of pressure

- Absolute pressure $p_{\text{abs}}$
- Atmospheric pressure $p_{\text{amb}}$
- Differential pressure $\Delta p$
- Gauge pressure $p_g$

\[ p_g = p_{\text{measured}} - p_{\text{amb}} \]

Fluctuation of $p_{\text{amb}} \sim \pm 5\%$

Direct measuring pressure instruments

Indirect measuring pressure instruments

- Using a liquid column
- Using the effect of a pressure acting on a material or on bodies of a certain shape

Electrical pressure sensors

The different types of pressure differ only with respect to their reference point.
Electrical Pressure Sensors

- Inductive Principle
  - LVDT - sensor
  - LVR - sensor

- Capacitive Principle

- Resistive Principle
  - Potentiometric Sensors

- Piezoelectric Effect
  - Hall effect

- Magnetic Field Sensors

- Metallic Strain Gauge
- Piezoresistive Sensors
- Strain Foile Gauge
- Thick-film Strain Gauge
- Gauge
Flow measuring instrument is defined as device which measures the flow rate of a fluid.

**Pressure difference across an orifice**

\[ Q = \alpha_D \cdot \frac{\pi \cdot d^2}{4} \sqrt{\frac{2 \cdot (p_1 - p_2)}{\rho}} \]

\( \alpha_D \)...flow discharge coefficient

\( \alpha_D \approx 0.7 \div 0.55 \)

Should not be used for determination of steady state characteristics!

**Displacement of a spring loaded floated element**
Flow meter – device which directly indicates the rate of flow of a fluid

- **Displacement principle**
- **Hydrodynamic principle**

Using kinetic energy of a fluid to drive a rotating system of blades (an impeller), whereas the rotational speed of the rotor is measured with an electric speed sensor (frequency measurement device)

- **Gear flow meter**
- **Screw flow meter**
- **Turbine**
Διαδικάσια μέτρησης

1. Keep \( p_1 \) and \( \theta_1 \) constant
2. Adjust different pressure levels at constant shaft speed
3. Record all measured values under steady state conditions

- Repeat measurements for different speed settings
- In case of variable displacement pump repeat measurements for 75%, 50% and 25% of \( V_{\text{max}} \)
- Repeat measurements for different temperatures \( \theta_1 \)

Provide table with measurement results

<table>
<thead>
<tr>
<th>No.</th>
<th>( n ) [RPM]</th>
<th>( T ) [Nm]</th>
<th>( p_1 ) [bar]</th>
<th>( \theta_1 ) [°C]</th>
<th>( p_2 ) [bar]</th>
<th>( Q_2 ) [l/min]</th>
<th>( \theta_2 ) [°C]</th>
<th>( p_{Se} ) [bar]</th>
<th>( Q_{Se} ) [l/min]</th>
<th>( \theta_{Se} ) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2001.46</td>
<td>15.03</td>
<td>19.77</td>
<td>50.50</td>
<td>25.11</td>
<td>145.13</td>
<td>61.20</td>
<td>1.20</td>
<td>0.21</td>
<td>65.30</td>
</tr>
<tr>
<td>2</td>
<td>2001.63</td>
<td>68.78</td>
<td>19.79</td>
<td>50.30</td>
<td>69.65</td>
<td>144.02</td>
<td>60.30</td>
<td>1.25</td>
<td>0.43</td>
<td>66.00</td>
</tr>
<tr>
<td>3</td>
<td>2001.79</td>
<td>130.59</td>
<td>19.86</td>
<td>50.30</td>
<td>120.82</td>
<td>142.92</td>
<td>60.40</td>
<td>1.28</td>
<td>0.56</td>
<td>65.20</td>
</tr>
</tbody>
</table>
Χαρακτηριστικά steady state
Effective volume flow rate is reduced due to compressibility of the fluid:

\[ \int_B^C \frac{dV}{V} = \int_B^C \frac{1}{K_A} dp \]

\[ \Delta V_B = V_B \left( 1 - e^{-\frac{1}{K_A} (p_C - p_B)} \right) \]

\[ \Delta V_B = V_B \frac{\Delta p}{K_A} \]

\[ Q_{SK} = n \Delta V_B \]

with \( n \) being the pump speed.
Effective volumetric flow rate

\[ Q_e = Q_i - Q_S \]

\[ Q_i = V \quad n = \alpha \quad V_{\text{max}} \quad n \]

\[ Q_S = f(\Delta p, n, V, \theta) \]

\( \theta \) ... temperature
Effective mass flow at pump outlet $Q_{\text{me}}$

Loss component due to compressibility does not occur!
Ευχαριστώ για την προσοχή σας

Email:
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